# Lecture 2: Intertemporal theory of CA

## 1.5 Understanding CA: Fischer's intertemporal theory of consumption

This is the backbone of any modern theory of the consumption function. We present a stripped-down version of it.

Assumptions:

1. endowment economy (no production)
2. Consumers: live for two periods and maximize their lifetime utility function $U(C_1, C_2) = u(C_1) + \beta u(C_2)$,

   where $0 < \beta < 1$ is the discount rate and $u$ is the per period utility function with $u' > 0$ and $u'' < 0$.

3. Endowments: $Y_1$ units of the consumption good in the first period of life and $Y_2$ in the second one. The good is non-storable.

4. Endowments can be freely borrowed and lent at the real interest rate $r$ subject to solvency.

$$B_1 = S_1 = (Y_1 - C_1) \tag{11}$$

$$B_2 - B_1 = S_2 = (Y_2 + rB_1 - C_2) \tag{12}$$

**Solvency:** with finite lifetimes solvency means that agents cannot die with a positive stock of debt; i.e. $B_2 \geq 0$. If the marginal utility of consumption is positive it implies $B_2 = 0$.

- Solvency then requires $S_2 = -B_1 = -S_1$ or $C_2 - Y_2 = (1+r)(Y_1 - C_1)$ or

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{13}$$

- The PDV of lifetime consumption equals the PDV of lifetime income (**intertemporal budget constraint (IBC)**).

- Intertemporal trade (borrowing and lending) decouples *current* consumption from *current* income (**consumption smoothing**) but not over lifetime.

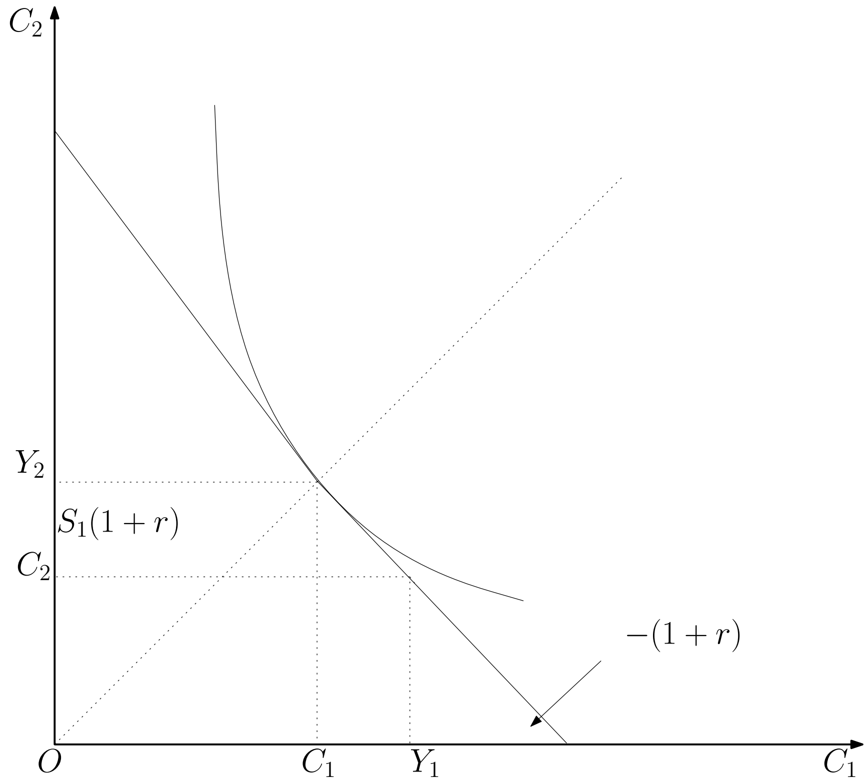
So consumers maximize $U(C_1, C_2) = u(C_1) + \beta u(C_2)$

$$\text{subject to the IBC} \qquad C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}. \tag{14}$$

- $\frac{1}{1+r}$ is the relative price of $C_2$ in terms of $C_1$ (market rate of exchange between the two).
- $\frac{\beta u'(C_2)}{u'(C_1)}$ is the marginal rate of substitution between $C_2$ and $C_1$ (subjective rate of exchange).
- At an optimum (max utility s.t. constraint) the two must be equal

$$\text{FOC} \quad \frac{\beta u'(C_2)}{u'(C_1)} = \frac{1}{1+r} \tag{15}$$

and the IBC (14) must hold.



$C_2$
$Y_2$
$S_1(1+r)$
$C_2$
$-(1+r)$
$O$
$C_1$
$Y_1$
$C_1$

The FOC can be rewritten as

$$\frac{u'(C_1)}{u'(C_2)\,.} = \beta(1+r)\,. \tag{16}$$

## Consumption profile

Consumption profile (ratio $C_2/C_1$) depends on $\beta(1+r)$ alone:

- $\beta(1+r) > 1$ implies $u'(C_1) > u'(C_2)$ or $C_2 > C_1$.
- $\beta(1+r) < 1$ implies $u'(C_1) < u'(C_2)$ or $C_2 < C_1$.
- $\beta(1+r) = 1$ implies $u'(C_1) = u'(C_2)$ or $C_2 = C_1$.

**Consumption level**

Assume $\beta\,(1+r) = 1$ (simplest case).

Use the FOC ($u'(C_1) = u'(C_2) \rightarrow C_1 = C_2$) to replace in IBC

$$C_1 + \frac{C_1}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{17}$$

or

$$\frac{2+r}{1+r}C_1 = \left(Y_1 + \frac{Y_2}{1+r}\right) \tag{18}$$

which implies

$$C_1 = \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r}\right) \sim \frac{1}{2}\left(Y_1 + \frac{Y_2}{1+r}\right).$$

- With decreasing marginal utility of consumption, consumers borrow and lend to smooth the consumption profile over time (roughly half in each period).

- Consumption depends on lifetime income (rather than current income **unlike Keynesian theory of consumption**). Changes in the time profile of income that leave its present value unchanged do not affect consumption.

- Saving is a buffer to smooth consumption in the fact of non-smooth endowment

$$S_1 = Y_1 - C_1 = \frac{Y_1}{2+r} - \frac{Y_2}{2+r}. \tag{19}$$

$S_1 > 0$ if $Y_1 > Y_2$ and $S_1 < 0$ if $Y_2 > Y_1$.

### 1.5.1 Changes in endowments (pure income effect)

$$C_1 = \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r}\right) \tag{20}$$

1. Consider a temporary endowment shock ($\Delta Y_1 > 0$ and $\Delta Y_2 = 0$. From $E^1$ to $E^2$.).

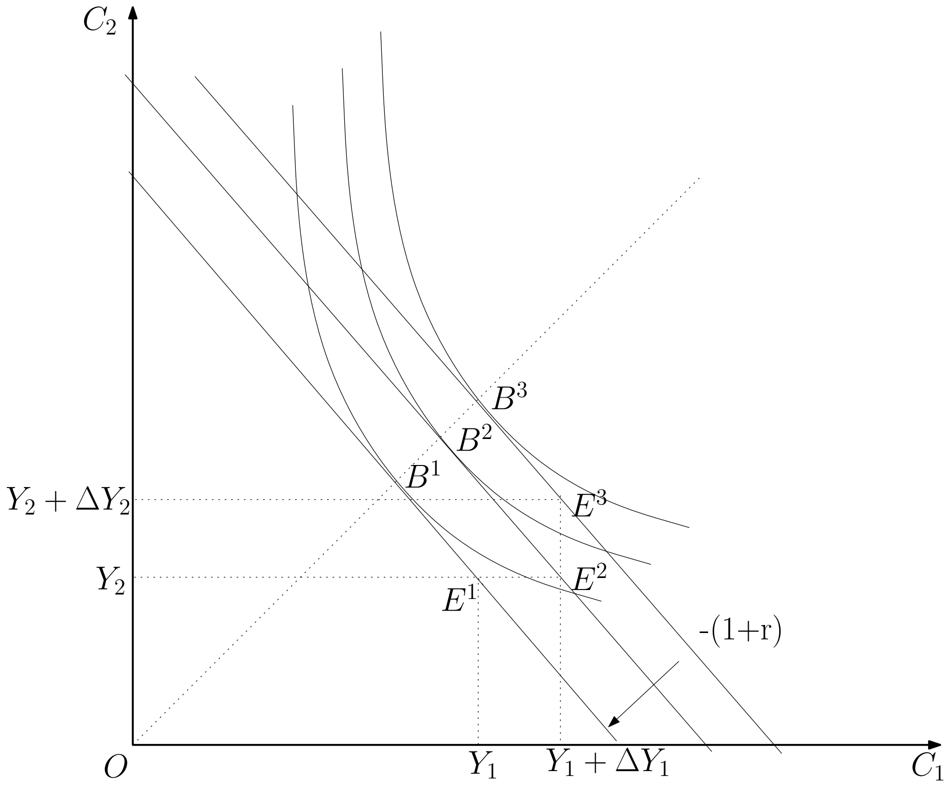
$$\Delta C_1 = \frac{1+r}{2+r}\Delta Y_1 < \Delta Y_1. \tag{21}$$

2. Consider a permanent endowment shock ($\Delta Y_1 = \Delta Y_2 > 0$. From $E^1$ to $E^3$.).

$$\Delta C_1 = \frac{1+r}{2+r}\left(\Delta Y_1 + \frac{\Delta Y_1}{1+r}\right) = \Delta Y_1 \tag{22}$$

Individuals fully adjust to permanent shocks, but save/dissave to smooth temporary ones.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$B^3$
$B^2$
$B^1$
$Y_2 + \Delta Y_2$
$E^3$
$Y_2$
$E^2$
$E^1$
-(1+r)
$O$
$Y_1$
$Y_1 + \Delta Y_1$
$C_1$

## 1.6 From individual saving to the current account

Assume identical consumers (one for simplicity) within a country (representative agent). Two periods.

### 1.6.1 Closed economy

- No borrowing/lending is possible as no gains from exchange (identical consumers).
- $C_i = Y_i$, $S_i = 0$ $i = 1, 2$.
- $\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1+r^a}$.

  Autarky interest rate $r^a$ : MRS at original endowment point. Price of $C_2$ relative to $C_1$ at which consumers are indifferent between consuming and saving the marginal unit.

### 1.7 Small open economy

Opening the economy to international trade across time allows borrowing/lending. Current account is the counterpart of saving.

Assumptions:

1. Small open economy. Does not affect world interest rate $r$.

2. No government sector, no investment.

$$CA_1 = S_1 = B_1 = \underbrace{Y_1 - C_1}_{NX_1} \tag{23}$$

$$CA_2 = B_2 - B_1 = S_2 = \underbrace{Y_2 - C_2}_{NX_2} + rB_1 \tag{24}$$

- $CA_i > 0$ if saving is positive.

- Note the difference between CA and NX.

- Solvency $B_2 \geq 0$ ($= 0$ given positive marginal utility) implies

$$CA_1 + CA_2 = 0 = B_1 + CA_2 = B_1(1+r) + Y_2 - C_2 \text{ or} \tag{25}$$

$$B_1 = -\frac{NX_2}{1+r} \text{ or} \tag{26}$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{27}$$

  – Eq. (25): a country cannot obtain more resources from abroad than it will transfer back in the future (and has no incentive to do the opposite); equivalently

  – Eq. (26): a country's stock of foreign debt must be offset by a positive PDV of future trade surpluses; equivalently

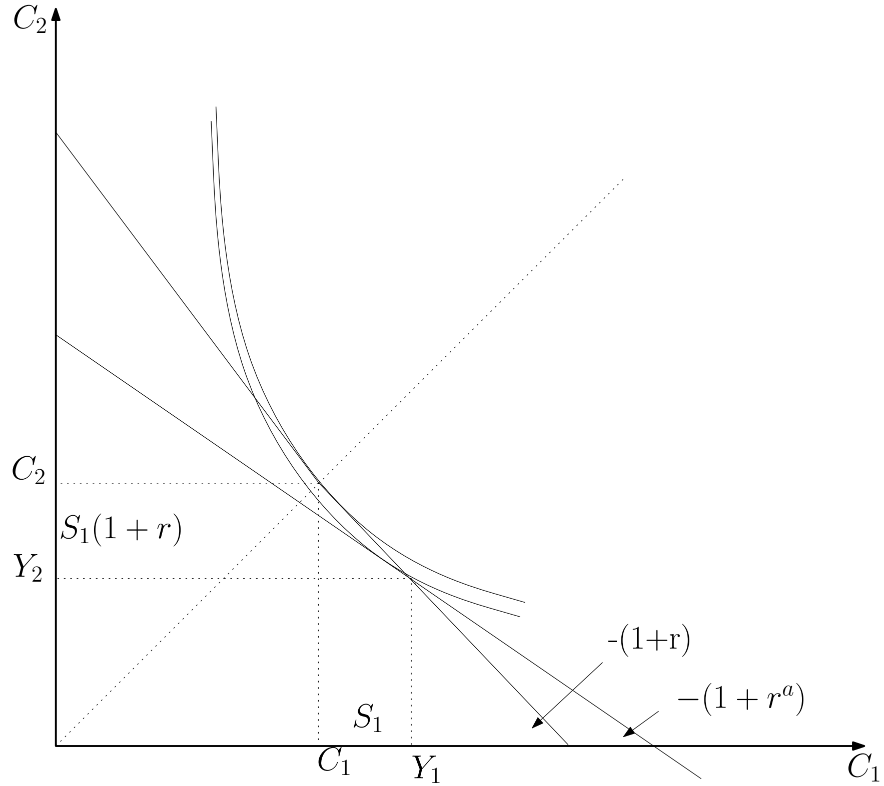
  – Eq. (27): IBC holds.

Otherwise the country is bankrupt (and unable to borrow today)!

Without no government expenditure or investment, $CA$ coincides with individual saving; i.e. determined by

$$\max_{C_1,C_2} u(C_1) + \beta u(C_2) \tag{28}$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{29}$$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## 1.8 Why do countries run CA surplus/deficits?

Equivalently: which countries borrow and which lend?

The intertemporal theory of the current account (consumption smoothing) predicts that

1. countries with high saving rates should run surpluses. What determines high saving rates? Present income high relative to future one.

   ex: Japan surplus because of ageing population, US deficit because of expectations of high future income.

2. Alternatively $CA \neq 0$ for countries such that $r^a \neq r$ (autarky interest rate different from world one).

   - $CA_1 < 0$ if $r^a > r$
   - $CA_1 > 0$ if $r^a < r$.

   In both cases there are **gains from trade** (country better off at the world interest rate if this is different from the autarky rate).

   It is suboptimal for countries to balance the current account at all times. The message of the intertemporal theory of the CA is that countries can improve their welfare by trading internationally across time.

3. Permanent versus temporary shocks. If a positive temporary shock save part of it ($\Delta CA > 0$), negative temporary borrow ($\Delta CA < 0$). Permanent shocks cut consumption.

$$C_1 = \frac{1+r}{2+r}\left(Y_1 + \frac{Y_2}{1+r}\right). \tag{30}$$

Finance temporary shocks, adjust to permanent one.

ex.: Oil shocks (terms of trade), earthquakes.

4. Governments: uneven patterns of taxes over time. Assume balance budget $T_i = G_i$.

   Endowments are now net of taxes $Y_i^* = Y_i - T_i$ .

$$CA_1 = Y_1 - T_1 - C_1 + T_1 - G_1 = Y_1 - G_1 - C_1 \tag{31}$$

   The rest follows. Note that given balanced budget $T_1 = G_1$ national saving coincides with private saving. Uneven patterns of taxes/expediture over time alter the current account.

   Wars (temporary increases in expenditure) should result in $CA$ deficits.