ECN 209 International finance

Mathematical results used in the course

This handout summarizes a few mathematical results used in the course.

1. Chain rule of differentiation. Given a function $f = f(x)$ and a function $g = g(f(x))$ the derivative of $g(f(x))$ with respect to x is

$$\frac{dg(f(x))}{dx} = g'(f(x))\, f'(x)\,. \tag{1}$$

This was used to solve the consumer problem in the intertemporal theory of the current account; i.e.

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \tag{2}$$

$$\text{s.t. } c_1 + \frac{c_2}{1+r} = y_1 \tag{3}$$

This is equivalent to

$$\max_{c_1, c_2} u(c_1) + \beta u(c_2) \tag{4}$$

$$\text{s.t. } c_1 = y_1 - \frac{c_2}{1+r} \tag{5}$$

and replacing for c_1

$$\max_{c_2} u\left(y_1 - \frac{c_2}{1+r}\right) + \beta u(c_2)\,. \tag{6}$$

The first term is the utility of c_1 which is a function of c_2 given that c_1 is related to c_2 by the budget constraint in equation 5. The first order condition obtained in the notes follows from the application of the chain rule to the above equation.

2. Logarithm of ratios/products.

$$\log(AB) = \log A + \log B \tag{7}$$

$$\log\left(\frac{A}{B}\right) = \log A - \log B \tag{8}$$

3.

$$\log(1+x) \simeq x \tag{9}$$

if x is small (close enough to zero) where $\simeq$ stands for approximately equal.

Proof: This can be easily derived by expanding $\log(1+x)$ in Taylor series around zero and dropping terms of order higher than one (otherwise just take my word for that).

4. The percentage change in the level is approximately equal to the difference of logarithms.

$$\frac{E_{t+1} - E_t}{E_t} \simeq e_{t+1} - e_t \tag{10}$$

where small letters denote logs (e.g. $e_t = \log E_t$).

Proof: the above can be rewritten as

$$\frac{E_{t+1} - E_t}{E_t} \simeq \log\left(\frac{E_{t+1}}{E_t}\right) \tag{11}$$

or

$$\frac{E_{t+1} - E_t}{E_t} \simeq \log\left(1 + \frac{E_{t+1} - E_t}{E_t}\right). \tag{12}$$

If you call x the term $(E_{t+1} - E_t)/E_t$, the result follows from point 3 above.