Homework 1 (aka problem set 5)

This problem set is to be posted in the coursework box (opposite the third-year notice board in the Economics Department) by 7 o'clock p.m. Monday 11 February. Your answers to it will be marked and will count for half of your total coursework mark. You are encouraged to work on the problem set in groups, but the final writing up has to be carried out individually. In the case of identical answers the mark will be split equally among the individuals involved (e.g. if two people provide an identical solution and it is worth 70 points each of them receives a mark of 35!)

1. Consider a small country inhabited by a total number of 1 identical citizens with a two-period lifetime and preferences over consumption of coconuts given by

$$U(c_1, c_2) = \sqrt{c_1} + \sqrt{c_2}, \tag{1}$$

with c_1 and c_2 denoting respectively first and second-period consumption. All quantities are measured in units of coconuts.

If the country accesses the international capital market it can borrow or lend at interest rate (measured in units of coconuts today per coconut tomorrow) $r = 0$. Individual endowments in the two periods are respectively $y_1 = 100$ and $y_2 = 105$. The country's net foreign asset position at the beginning of period 1 is $B_0 = 0$.

   (a) What is the relative price of c_2 in terms of c_1? Explain why in equilibrium B_2, the stock of assets at the end of period 2, has to equal zero. Write down the intertemporal budget constraint for the country in question.

   (b) Note that equation (1) implies that β, the consumer's subjective rate of discount, equals 1. Solve the consumer problem and derive the value of consumption and the current account surplus or deficit in period one. Given your finding, discuss whether the autarky interest rate in the country in question is higher or lower than the world interest rate.

   (c) Suppose the first period endowment increases by $\Delta y_1 = 10$ with no change in the second period. Derive the change in the first period current account.

   (d) Suppose endowments increase in both periods by $\Delta y_1 = \Delta y_2 = 10$. Derive the change in the first-period current account. Is the change larger or smaller (in absolute value) than in point (c)? What is the economic intuition behind the difference (if any) between the current account response in point (c)?

2. Consider a two-country world (home and foreign). The foreign country consumes only tradables. The home country, instead, consumes both tradables and non-tradables. The share of non-tradables in the home consumption basket is 0.5. The production function for tradables is identical in both countries and is given by $Y_T = a_T L_T$, where Y_T is tradable output, L_T the associated labour input and $a_T > 0$, its productivity. The home production function for non-tradables is $Y_N = a_N L_N$, where Y_n is non-tradable output, L_N the associated labour input and $a_N > 0$, its productivity. Both product and labour markets are perfectly competitive and labour is perfectly mobile within a country.

(a) Assuming that the Law of One Price holds for tradables, derive an expression for the real exchange rate between the home and the foreign country as a function of the relative productivity in the two sectors.

(b) Suppose that both a_T and a_N grow at 3%. What is the rate of growth of the real exchange rate? Do absolute or relative PPP hold? Explain your findings.

(c) Suppose that a_T grows at 4% while a_N grows at 3%. What is the rate of growth of the real exchange rate? Do absolute or relative PPP hold? Explain your findings.