International finance
Solution to problem set 4

1. Under the assumptions uncovered interest parity holds; i.e.

$$(1+i_t) = (1+i_t^*)\,\frac{E_{t+1}^e}{E_t} \tag{1}$$

Explain why (no arbitrage). Alternatively UIP can be written as

$$(1+i_t) = (1+i_t^*)\left(1+\frac{E_{t+1}^e - E_t}{E_t}\right) \tag{2}$$

(a) Time t is measured in years. $\frac{E_{t+1}^e - E_t}{E_t} = 0.1 \cdot 0.1 = 0.01$ Taking logs the interest rate differential is approximately given by

$$\dot{i}_t - i_t^* \simeq \frac{E_{t+1}^e - E_t}{E_t} = 0.01 \tag{3}$$

(b) Time t is measured in months. So the expected depreciation is now 0.01 but over one month. So the interest rate differential is 0.01 but on a monthly basis.

$$\frac{(1+i_t)}{(1+i_t^*)} = 1 + 0.01 \tag{4}$$

To transform monthly/daily interests into yearly rates, one compounds them; $(1+i_t^a)^{1/n} = \left(1+i_t^j\right)$ where i_t^j is the monthly/ daily rate, i_t^a is the annual rate and n is the number of months/days in a year. So if i_t^j is the monthly rate it is

$$(1+i_t^a) = \left(1+i_t^j\right)^{12} \tag{5}$$

and

$$(1+i_t^a) = \left(1+i_t^j\right)^{365} \tag{6}$$

if i_t^j is the daily rate.
So the annualized rate differential is

$$\frac{(1+i_t^a)}{(1+i_t^{a*})} = 1.01^{12} \tag{7}$$

which implies

$$\dot{i}_t - i_t^* \simeq 0.127 \tag{8}$$

(c) Time t is measured in days. So the expected depreciation is now 0.01 but over one day.

$$\frac{(1+i_t^a)}{(1+i_t^{a*})} = 1.01^{365} = 37.37 \tag{9}$$

which implies (note that the log approximation is not very good given the magnitudes involved

$$\dot{i}_t - i_t^* \simeq 36.37 \tag{10}$$

or 3637%. Mention overnight interest rates of 1500% (annualized) in Sweden at the time of the collapse of the European monetary system $\rightarrow$ even small expected depreciation over a very short horizon becomes a very high rate annualized.

The part of the question about the general formula has a typo. It should have t (rather than τ) measured in months. Months is the unit in which time is measured and τ is the relevant duration in months over which the depreciation is calculated.

The expected depreciation gives the interest rate differential over duration τ. There are $12/\tau$ periods $\tau-$months long in a year and therefore

$$(1+i_t^a)^{\tau/12} = (1+i_t^{a*})^{\tau/12}\left(1+\frac{E_{t+\tau}^e - E_t}{E_t}\right). \tag{11}$$

Taking logs and approximating

$$i_t^a - i_t^{a*} \simeq \frac{12}{\tau}\left(\frac{E_{t+\tau}^e - E_t}{E_t}\right). \tag{12}$$

So for a given expected depreciation the required *annualized* interest rate differential is larger the smaller is τ.

2. Since the foreign exchange risk premium is small in narrow target zones, we can assume it is zero with little loss of generality. Covered interest parity implies that the forward premium over a certain maturity τ equals the interest rate differential over the same maturity

$$\frac{(1+i_t^\tau)}{(1+i_t^{\tau*})} = \frac{F_t^\tau}{E_t} = 1 + \frac{F_t^\tau - E_t}{E_t} \tag{13}$$

where F_t^τ is the forward exchange rate for delivery τ from today and the fraction is the forward-premium (percentage excess of forward rate over spot one). With no risk premium UIP has to hold, which implies

$$\frac{E_{t+\tau}^e}{E_t} = \frac{(1+i_t^\tau)}{(1+i_t^{\tau *})} \tag{14}$$

and

$$\frac{E_{t+\tau}^e}{E_t} = \frac{F_t^\tau}{E_t}. \tag{15}$$

Under these assumptions a forward premium of 0.07 over a given maturity τ implies an expected depreciation of 0.07 over the same maturity which implies that the market does not believe that the exchange rate will stay within a band of total width equal to 0.05. The target zone is not credible over that maturity.