International finance
Solution to problem set 5 (aka Homework I)

1. (a) The relative price of c_2 in terms of c_1 is $1/(1+r) = 1$ given r=0.
   The changes in the stock of net foreign assets $B_i,\ i = 1, 2,$ in period 1 and 2 are respectively

$$B_1 - B_0 = rB_0 + y_1 - c_1 \tag{1}$$
$$B_2 - B_1 = rB_1 + y_2 - c_2. \tag{2}$$

   Given finite horizon, solvency requires $B_2 \geq 0$ (the individual/country cannot die in debt). Furthermore, since the marginal utility of consumption is always positive $B_2 = 0$ (it is not optimal to die with a positive stock of assets).
   Imposing $B_2 = 0$ in (2) and using it to replace for B_1 in (1) one obtains the intertemporal budget constraint (IBC)

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} + (1+r)B_0. \tag{3}$$

   or given $r = 0$

$$c_1 + c_2 = y_1 +_2 . \tag{4}$$

   PDV of lifetime consumption equals PDV of lifetime income.

   (b) Optimality requires the marginal rate of substitution to equal the price ratio. Namely,

$$\frac{MU_{c_2}}{MU_{c_2}} = \frac{\frac{1}{\sqrt{c_2}}}{\frac{1}{\sqrt{c_1}}} = \frac{1}{1+r} \tag{5}$$

   or equivalently $c_2 = c_1$.
   Replacing for c_2 in the intertemporal budget constraint and solving for c_1 gives

$$c_1 = \frac{y_1 + y_2}{2} = 102.5. \tag{6}$$

   Desire to smooth consumption (concave preferences) implies that lifetime income is split equally across periods.
   The current account balance equals the excess of the country's total income over its consumption (saving) and equals the change in the current stock of net foreign assets. From equation

$$CA_1 = B_1 - B_0 = y_1 - c_1 = \frac{y_1 - y_2}{2} = \frac{100 - 105}{2} = -2.5. \tag{7}$$

   The country borrows internationally to smooth consumption, given that the future endowment is higher than the present one.
   If the country borrows on the international capital market it has to be the case that the autarky interest rate is above the international one (this is all I required students to say).

One can prove this by solving for the autarky interest rate. Since optimality requires $c_i = y_i$ in autarky (positive marginal utility, coconuts are perishable and capital markets to reallocate them intertemporally) the autarky relative price of consumption equals the marginal rate of substitution at the endowment point; i.e.

$$\frac{1}{1+r^A} = \frac{\sqrt{y_1}}{\sqrt{y_2}}, , \tag{8}$$

where rA is the autarky interest rate. It follows that $1 + r^a = \sqrt{1.05}$ which implies that $r^A > r = 0$.

(c) Using equation (7)

$$\Delta CA_1 = \frac{\Delta y_1 - \Delta y_2}{2} = 5. \tag{9}$$

Starting from an optimum where consumption is smooth (the same in both periods), the country saves half of the income increase in the first period to increase consumption in both periods by the same amount.

(d) The same equation (9) implies $\Delta CA_1 = 0$ as the increase in the endowment is permanent (already smooth across time).

2. (a) Denote respectively by P and P^* the prices of the home and foreign consumption baskets measured in the respective country currency. It is $P^* = P_T^*$ as the foreign country consumes only tradables. Instead, the price of the home basket is $P = P_T^{1-\alpha} P_N^{\alpha}$, with $\alpha = 0.5$.

The real exchange rate is the relative price of the two consumption baskets. If one denotes by E the nominal exchange rate (expressed as price of the foreign currency in units of the home one) the real exchange rate is

$$RER = \frac{EP^*}{P} = \frac{EP_T^*}{P_T^{1-\alpha} P_N^{\alpha}} = \frac{EP_T^*}{P_T} \left(\frac{P_T}{P_N}\right)^{\alpha}. \tag{10}$$

Since the law of one price holds for tradables the first factor to the right of the last equality equals one (home and foreign price of tradables in the same currency are equal) and

$$RER = \left(\frac{P_T}{P_N}\right)^{\alpha} \tag{11}$$

and is a function of the home relative price of tradables versus non-tradables. This is the quantity we need to determine.

Free mobility of labour implies that the wage is the same in both sectors.

Profits in the two sectors are given by $P_i a_i L_i - wL_i,\ i = T, N$. Perfect competition requires zero profits or $P_i a_i = w$, which implies

$$\frac{P_T}{P_N} = \frac{a_N}{a_T}. \tag{12}$$

Replacing in the equation for the real exchange rate

$$RER = \left(\frac{a_N}{a_T}\right)^{\alpha}. \tag{13}$$

Absolute PPP requires the real exchange rate to equal one (price of consumption baskets to be equalized across countries). This is highly unlikely given non-tradables. Relative PPP just requires RER to be constant.

Using properties of logs, the rate of growth of the real exchange rate

$$\frac{\Delta RER}{RER} = \alpha \left(\frac{\Delta a_N}{a_N} - \frac{\Delta a_T}{a_T} \right) \tag{14}$$

(b) $\left(\frac{\Delta a_N}{a_N} - \frac{\Delta a_T}{a_T} \right) = 0$. RER is constant and relative PPP holds. One cannot say whether absolute PPP holds without knowing a_i.

(c) $\frac{\Delta RER}{RER} = \alpha(0.03 - 0.04) = 0.5 \times (-0.1) = -0.05$. Relative PPP (hence absolute PPP) does not hold as the relative price of tradables versus non-tradables is not constant.