International finance
Solution to problem set 8

1. Please refer to the model in the lecture notes i.e.

$$\text{LM: } m_t - p_t = \bar{y} - \alpha i_t \tag{1}$$
$$\text{PPP: } e_t = p_t - p_t^* \tag{2}$$
$$\text{UIP: } i_t = i_t^* + \Delta e_{t+1} \tag{3}$$

Normalize, without loss of generality, i_t^*, p^* and $\bar{y}$ to zero. Replacing using PPP and UIP in LM, we obtain:

$$m_t - e_t = -\alpha \Delta e_{t+1}. \tag{4}$$

Left hand side is real money supply. Right hand side is real money demand. This is crucial for the intuition.

Under a *credible* fixed exchange rate system it is $\Delta e_{t+1} = 0$ and $e_t = \bar{e}$. Hence, the nominal money supply $m_t = \log(D_t + R_t)$ has to stay constant which requires reserves to fall as D_t increases to keep the total constant.

After reserves are exhausted, $m_t = \log D_t$ and the exchange rate floats $\Delta e_{t+1} = \Delta p_{t+1} = m_{t+1} - m_t = \log D_{t+1} - \log D_t$.

So when reserves are exhausted *and* the peg is abandoned, the exchange rate follows

$$e_t = \log D_t + \alpha(\log D_{t+1} - \log D_t). \tag{5}$$

This is the *shadow* floating exchange rate. It tells you the value of the exchange rate once reserves are exhausted and the exchange rate float.

The shadow floating exchange rate equals $e_t = \log D_t + \alpha\mu$ under the first scenario and $e_t = \log D_t$ after the second one.

Draw the two in a diagram like the one in the lecture. The first line crosses the $e_t = \bar{e}$ line at a lower value of $\log D_t$ (the peg collapses earlier). The nominal exchange rate cannot jump in a foreseen way (no arbitrage, see lecture notes) at the time of the collapse. So it has to equal $\bar{e}$ at T the time of the collapse. So the collapse takes place at the level of domestic credit where the shadow floating exchange rate equals $\bar{e}$. Stress to students that that is the way to solve for D_T.

(a) Noticing that $\bar{e} = \log(D_t + R_t)$ and that $\bar{e}$ equals the shadow floating exchange rate at the time of the collapse you can solve for the size of reserves immediately before the collapse (the size of the speculative attack); i.e.

$$\log(D_{T_1} + R_{T_1}) = \log D_{T_1} + \alpha\mu \tag{6}$$

or $R_{T_1} > 0$ in the first scenario and

$$\log(D_{T_2} + R_{T_2}) = \log D_{T_2}, \tag{7}$$

or $R_{T_2} = 0$ in the second scenario. Note that the level of domestic credit, hence the time of the collapse is different, hence the different indeces.

You can draw the time path of reserves (see Figure in lecture notes). In the second case, they go to zero smoothly. In the first one, they jump discontinuously down to zero at time T.

For the intuition, see equation (4). In the first case Δe_{t+1} jumps discontinuously from zero to μ after the collapse of the peg. In the second case, it stays at zero as domestic credit stop growing. Hence, real money demand (the RHS of (4)) jumps down at the moment of the collapse in the first case but not in the second one. For money market equilibrium to be maintained, the real money supply has to do the same. Since e_t (hence p_t given PPP) cannot jump, the nominal money supply has to do the adjustment. Hence, it has to jump down (stock shift depletion of reserves) in the first case but not in the second one.

(b) Use the picture you have drawn with the fixed exchange rates and the two shadow floating exchange rates as a function of $\log D_{t.}$ Consider the scenario in which the attack is postponed relative to the first case; i.e. it takes place at some level of domestic credit $D_{T_1'}$ between D_{T_1} and D_{T_2}. If investors find out they are in the second scenario. The float exchange rate is below the fixed one. The money supply is lower than it is necessary to support the peg. The central bank can still support the peg, since what it has to do is just to buy reserves to expand the money supply (rather than sell them to keep them constant which she can only do until it hits $R_t = 0$). So the exchange rate does not jump in this scenario. Yet, if scenario one turns out to be correct the exchange rate jumps up to the shadow floating one and investors make a capital loss. Hence, in expected value delaying the attack beyond D_{T_1} entails a capital loss as long as the first scenario happens with positive probability. Faced with the uncertainty, investors should attack at D_{T_1} since they lose nothing from attacking if the peg turns out to be still viable, but they lose from not attacking in the case it is not. Attacking at T_1 is a one-sided bet. You cannot lose.