LECTURE 6

# Real busyness cycle theory

## 1 Business cycle stylized facts.

- Business cycles: fluctuations of output around trend (relative high frequency phenomenon, 3 to 5 years).
- Output fluctuations are well captured by linear process subject to white noise shocks. E.g. AR(2)
$$y_t = \gamma_0 + \gamma_1 y_{t-1} + \gamma_2 y_{t-2} + \epsilon_t \tag{1}$$

  Therefore reasonably simple models may be able to generate plausible dynamics.
- Detrend the data.
- Facts for the US economy.
- Quantities:

1. Consumption strongly +vely correlated with GNP but roughly half as volatile as output.
2. Investment strongly +vely correlated with GNP and roughly 3 times as volatile as output.
3. Total hours worked and employment strongly procyclical. Total hours fluctuate nearly as much as output. Most fluctuation in hours reflect fluctuations in employment.
4. Average labour productivity somewhat procyclical and fluctuates significantly less than output (correlation between output and hours less than one).
5. Solow residual highly variable and strongly +vely correlated with GNP. With Cobb Douglas production $Y_t = A_t K_t^{\alpha} (h_t L_t)^{1-\alpha}$, where $h_t$ are hours per worker the log of the Solow residual can be obtained as

$$\log A_t = \log Y_t - \alpha \log K_t - (1-\alpha)\left(\log h_t + \log L_t\right). \tag{2}$$

A significant part of output fluctuation is unaccounted for by fluctuations in inputs.

6. Capital stock fluctuates much less than output and largely uncorrelated.
7. Government purchases roughly uncorrelated with output.

- Prices:

1. Real wage mildly procyclical in establishment survey and not very variable. Acyclical in aggregate data.
2. Real interest rate mildly countercyclical.
3. Inflation strongly procyclical.
4. Money and nominal interest rates positively correlated with output.

## 2 RBC: Ramsey with shocks

- The people who first advocated the approach (Kydland and Prescott) got the Nobel prize in 2004.
- Growth and business cycle as unified phenomenon.
- Can a Walrasian (competitive) set up explain fluctuations.

- Need shocks: productivity (Solow residual) and government shocks. Originally RBC assumed Solow residual main source of shocks.
- Only real shocks. No role for money in Walrasian setup without some form of market incompleteness.
- Once we introduce uncertainty the Ramsey model is much more difficult to solve. The equilibrium is now a sequence of random variables (a stochastic process). Possible solutions:
  - Study unanticipated one-off shocks.
  - Choose special functional forms or two period horizon. Guess and verify solution.
  - Log-linearize the model around steady state.
  - Dynamic programming and use a computer.

### 2.1 RBC with constant labour supply

- No population growth ($L_t = 1$) nor trend productivity growth.
- 100% depreciation (very restrictive).

Consider the following discrete-time version of the Ramsey model. We have argued that the decentralized solution coincides with the planning solution. For convenience we analyze the latter.

The production technology is

$$Y_t = A_t K_t^{\alpha} (h_t L_t)^{1-\alpha} \tag{3}$$

where $h_t$ is hours worked per worker. Here, $h_t$ is exogenously given and independent of time.

At each time $t$ the consumer solves the problem

$$\max_{[C_{t+i}, B_{t+i+1}]_{i=0}^{\infty}} \sum_{i=0}^{\infty} \frac{E\left[\log C_{t+i} | I_t\right]}{(1+\rho)^i} \tag{4}$$

$$\text{s.t.} B_{t+i+1} = W_t h_t + B_{t+i}(1 + r_{t+i}) - C_{t+i} \tag{5}$$

The consumer chooses a sequence at time $t$ but at time $t+1$ he reoptimizes if a new shock takes place. Replace for $C_{t+i}$ using the accumulation constraint to obtain

$$\max_{[B_{t+i+1}]_{i=0}^{\infty}} \sum_{i=0}^{\infty} \frac{E\left[\log\left(W_{t+i}(1-l_{t+i})+B_{t+i}(1+r_{t+i})-B_{t+i+1}\right)|I_t\right]}{(1+\rho)^i}, \tag{6}$$

where $1-l_t=h_t$

The intertemporal optimality condition (Euler equation) at time $t$ is the FOC with respect to $B_{t+1}$

$$\frac{1}{C_t}=\frac{1}{1+\rho}E\left[\frac{1+r_{t+1}}{C_{t+1}}|t\right]. \tag{7}$$

We cannot take the interest rate in out of the expectation but otherwise the intuition is the same (The problem is $r_{t+1}$ not $B_{t+1}$ which is chosen at time $t$ hence non-stochastic).

Impose equilibrium, noticing that FOC for capital is

$$\alpha A_{t+1}K_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}=1+r_{t+1} \tag{8}$$

to obtain the Euler equation

$$\frac{1}{C_t} = \frac{1}{1+\rho} E\left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1}}{C_{t+1}} | t\right], \tag{9}$$

where we have normalized $h_{t+1}$ to one without loss of generality.

In equilibrium the dynamic constraint becomes

$$K_{t+1} = A_t K_t^{\alpha} - C_t. \tag{10}$$

The last two equations are the discrete time counterpart of the $\dot{c}$ and $\dot{k}$ equations in the continuous time Ramsey model.

- Economic intuition. All the action comes from Euler equation. Suppose the economy is initially in steady state.

  **Example 1:** Consider a permanent shock that does not affect the MPK (e.g. government expenditure). Consumption smoothing requires one-off permanent adjustment in consumption, but no change in $K$.

  **Example 2:** Consider a permanent, productivity shock. It raises output today and tomorrow at unchanged capital stock. Same effect as before (consumption smoothing), but it also raises MPK tomorrow (consumption tilting). Euler equation implies upward sloping consumption profile hence capital stock must be growing and positive net investment. In general consumption may go up or down depending on whether consumption smoothing prevails or not over consumption tilting.

If shock is transitory, consumption smoothing effect is smaller and, since output is given, investment is higher today, but less persistent.

Problem: stochastic difference equation, in general no closed-form solution. In this case, guess a solution for the consumption function and verify.

In general, a solution for consumption at time $t$ must be a function of the whole information set $I_t$ (i.e. the whole history of past and present shocks until time $t$.). Easier if shocks are not too persistent (only a subset of their history helps forecasting).

Guess $C_t = \beta A_t K_t^\alpha$ with $\beta$ to be determined. Replace in the Euler equation to obtain

$$\frac{1}{\beta A_t K_t^\alpha} = \frac{1}{1+\rho} E\left[\frac{\alpha A_{t+1} K_{t+1}^{\alpha-1}}{\beta A_{t+1} K_{t+1}^\alpha} | t\right] \tag{11}$$

or

$$\frac{\alpha}{1+\rho} A_t K_t^\alpha = K_{t+1}. \tag{12}$$

Replacing in the dynamic constraint we obtain $\beta = 1 - \alpha/(1+\rho)$ Log utility implies both consumption and saving are proportional to current income (as in Solow) independently of expectations of future $A_{t+i}$.

Taking logs (denoted by lower case letters) of equation (12)

$$k_{t+1} = b + \alpha k_t + a_t, \tag{13}$$

with $b = \log[\alpha/(1+\rho)]$.

- First-order difference equation in $k$.
- If $a_t$ is white noise, persistence of $k$ is $\alpha$, the capital share (because of full depreciation).

- Noticing that $y_t = \alpha k_t + a_t$ the same applies for output as

$$y_{t+1} = b + \alpha y_t + a_{t+1}. \tag{14}$$

- If instead $a_{t+1} = \lambda a_t + \epsilon_{t+1}$ (AR(1) ) we get

$$y_{t+1} - \lambda y_t - b(1-\lambda) - \alpha(y_t - \lambda y_{t-1}) = \epsilon_t. \tag{15}$$

- Second order stochastic difference equation for output. Good! But...

- The model has little *propagation*. It generates little extra persistence over and above that of the driving process $a_t$.

- The model has little *amplification*, if shock is white noise($\lambda = 0$) the variance of output is $\sigma_\epsilon^2/(1-\alpha^2)$ where $\sigma_\epsilon^2$ is the variance of the shock. With $\alpha = .3$ it is $1/(1-\alpha 2) \sim 1$.
  We need implausibly large fluctuations in technological progress (roughly 1% at quartely frequency).

- Consumption, investment and capital stock (the two coincide with 100% depreciation) as variable as output. This follows from constant saving rate and full depreciation.

- Introducing less than 100% implies that capital stock takes more time to adjust. Hence, MPK (decreasing in capital) falls less fast with capital accumulation, bigger consumption tilting effect, bigger respone of saving (hence investment) and smaller of consumption.

- Highly procyclical real wage as both capital and Solow residual are highly procyclical. With fixed labour, MPL fluctuates as much as output as $W_t = (1-\alpha)Y_t/L_t$. Wages are acyclical or mildly procyclical in the data. Productivity fluctuates less than output.

### 2.2 RBC with endogenous labour supply

- Above model cannot explain positive correlation between output and employment.
- Over long term correlation due to demographic changes (growth model), but positive correlation at business cycle frequency.
- Competitive market: unemployment is voluntary. So real wage must fluctuate a lot (counterfactual) or labour supply respond a lot to small fluctuations in real wage.
- Endogenize labour supply. Utility function now also depends on leisure.

$$U_t = \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} E\left[\left(\log C_{t+i} - \frac{\sigma}{\sigma+1}(1-l_{t+i})^{\frac{\sigma+1}{\sigma}}\right) | I_t\right]. \qquad (16)$$

Replacing for $C_{t+i}$ using dynamic constraint the consumer problem becomes

$$\max_{[B_{t+i+1},l_{t+i}]_{i=0}^{\infty}} \tag{17}$$

$$\sum_{i=0}^{\infty} \frac{E\left[\left(\log\left(W_{t+i}(1-l_{t+i})+B_{t+i}(1+r_{t+i+1})-B_{t+i+1}\right)-\frac{\sigma}{\sigma+1}(1-l_{t+1})^{\frac{\sigma+1}{\sigma}}\right)|I_t\right]}{(1+\rho)^i}, \tag{18}$$

The FOC for $B_{t+1}$ (Euler equation) is the same as before

$$\frac{1}{C_t}=\frac{1}{1+\rho}E\left[\frac{1+r_{t+1}}{C_{t+1}}|t\right], \tag{19}$$

but now there is also a FOC for $l_t$

$$-\frac{1}{C_t}W_t+(1-l_t)^{\frac{\sigma+1}{\sigma}-1}=0. \tag{20}$$

Intratemporal MRS between consumption and leisure equals the relative price of the two (real wage).

$$(1-l_t)^{\frac{1}{\sigma}}=\frac{W_t}{C_t} \tag{21}$$

Two effects.

- Substitution effect: Higher $W_t$ leads people to reduce leisure.
- An income/wealth effect. People are richer and want to increase both consumption and leisure if both are normal goods. Higher $C_t$ requires lower $1 - l$ Net effect depends on the strength of the two effects. Substitution (elasticity), and wealth (persistence). The more transitory the shock, the smaller the increase in C, and so the stronger the substitution effect. The more permanent, the stronger the wealth effect. Employment could decrease.

We can also think of leisure choice in intertemporal terms by using the Euler equation we can also write the intertemporal condition for labour supply

$$\frac{1-l_t}{1-l_{t+1}} = \left(\frac{W_t(1+r_{t+1})}{W_{t+1}(1+\rho)}\right)^{\sigma}. \tag{22}$$

- Relative labour supply depends on relative wage and interest rate.
- Permanent shock. Raises wages by the same amount in both periods at constant $K$ (hence, constant $r_{t+1}$.) No change in labour supply.

- Temporary shock. $W_t$ increases relative to $W_{t+1}$. Labour supply increases together with output. It responds the more the larger the intertemporal elasticity of labour substitution $\sigma$.

How does the model perform?

- With 100% depreciation badly. Hours worked are constant as before. Effect of rate of interest and relative wages cancel each other. Alternatively, income and substitution effect cancel out intratemporally as $C_t$ and $W_t$ go up by the same amount.
- With realistic depreciation $\delta = 0.07$, quite well though:
    1. Output fluctuates only 70% as much as in the US economy.
    2. Labour input fluctuates only 50% as much as in the US economy (wages still fluctuate too much).
    3. In other words the model still predicts a strong positive correlation between labour input and labour productivity. Hardly any in the data.

4. Consumption now fluctuates too little relative to output compared to the US economy.
5. Since shocks are very persistent, intertemporal labour substitution is driven by changes in interest rate more than relative wages. Interest rate increases a lot as capital is slow to adjust (low depreciation). Persistence is driven by persistent technological shocks and sluggish dynamics of capital.

Empirical implications

- Negative serial correlation of employment in response to wage shocks. Counterfactual. To obtain positive correlation we need cost of adjustment in labour.
- Empirical problems: intertemporal elasticity of substitution must be implausibly high (instead it is low in the data with most estimate between 0 and .45) and wages have little transitory component (most movements are permanent rather than temporary).
- Real interest has very little cyclicality in the data.
- If $W_t$ is unchanged as is the case in the data, the intratemporal optimality con-

dition implies that consumption and leisure must move together if both normal goods. Hence, negative correlation between consumption and labour supply. But this is at odds with the data.

So labour supply must be more elastic (ideally very close to infinitely elastic) so that it responds a lot to negligible changes in wages. Labour lotteries: fix hours $h_0$. With probability $\pi_t$ work $h_0$ hours and get a wage $w_t$. With probability $1 - \pi_t$ work zero hours and get $w_t$ anyway. The lottery provides full insurance against unemployment.

Assume instantaneous utility is

$$u(C_t, 1 - l_t) = \log C_t + \log(1 - h_t). \tag{23}$$

Its expected value is

$$u(C_t, 1 - l_t) = \log C_t + \pi_t \log(1 - h_0). \tag{24}$$

In equilibrium if we denote average hours per head by $h_t$ it is $\pi_t = h_t/h_0$ and it is

$$u(C_t, 1 - l_t) = \log C_t + \frac{h_t}{h_0} \log(1 - h_0), \tag{25}$$

which is linear in $h_t$. This corresponds to $\sigma = \infty$ in the above optimality condition for hours. Labour supply respond a lot (through changes in employment not hours) to negligible changes in wage.

## 3 Summing up

- In steady state Ramsey model coincides with Solow model (only the saving rate is endogenized). So the Ramsey models shares the same problems of the the Solow model.
- Need very variable and persistence technological shocks to generate persistence in output. Convergence to steady state too fast if measured capital share is the true one $\rightarrow$ Too little persistence of deviations from steady state.
- Consistent with large persistent component of output fluctuations.
- Price rigidities (keynesian models) eventually must die out as prices adjust.
- Labour indivisibility.

- Distortionary taxes, thus affecting the Euler equation.

**Criticisms**:

- Little amplification: so model must assume implausibly large deviation of Solow residual. Furthermore Solow residual is strongly correlated with a bunch of other variables.
- Recessions as negative productivity shocks? (absurd!).
- Too large elasticity of intertemporal substitution is necessary.
- No monetary rigidities. Misspecification error.

## Response to criticism

- The model does rather a good job at reproducing the statistical properties of macro variables. Whatever shocks are driving the cycle they must be such as to look like the Solow residual.
- New and promising research path: look for plausible sources of shocks (taxes, changes in union power and/or regulation) which have the same implications as Solow residual.

This is really exciting current research. It is very well summarized in "Business Cycle Accounting" by Chari, Kehoe and McGrattan, downloadable at http://woodrow.mpls.frb.fed.us/research/SR/SR328.pdf