Macroeconomics A

Problem set 3

1. Assume an economy in which there are two activities: production ideas or knowledge, A, and production of a final and intermediate good Y (our usual output that can either be consumed or invested to increase the capital). Output is produced using capital K, labor L and knowledge A according to the technology

$$Y = K^\alpha (A(1 - \alpha_L)L)^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$ and α_L is the constant proportion of workers employed in the output-producing sector. The exogenous rate of growth of the total labour force is n. In each period, a constant fraction s of the output Y is invested in new machines and the depreciation rate of the existing stock of machines is zero. New knowledge is produced using researchers and existing ideas according to the technology

$$\dot{A} = \delta(\alpha_L L)^\lambda A^\varphi, \tag{2}$$

where $0 < \lambda < 1$, $\varphi \geq 0$.

(a) Show that the marginal product of labour in the production of ideas is decreasing. What is the intuition behind it?

(b) Derive the steady state rates of growth of the stock of ideas A, capital per worker K/L and income per capita Y/L in this economy when $\varphi < 1$ and $n > 0$. What happens to these rates of growth when φ tends to one.

(c) Derive the steady state rates of growth of the stock of ideas A, capital per worker K/L and income per capita Y/L in this economy when $\varphi = 1$ and $n = 0$.

(d) What is the impact on the steady state rate of growth of an increase in the proportion α_L of workers employed in the research sector under case (b) and case (c) respectively. What on the out of steady state rate of growth of the stock of knowledge A? Comment on the other predictions of the two classes of models.

(e) Consider the production function for output Y. For which values of parameters does it display increasing, constant or decreasing returns to scale to *all factors which are not fixed in steady state* in the output production ? (Hint: to figure out which factor are not fixed in steady state use your findings in points (b) and (c) above.