Macroeconomics A

Problem set 4

1. (Taken from ex. 3.10 p. 168 in Romer). Assume an economy in which output is produced using capital K, labor L and knowledge A according to the technology

$$Y = K^{\alpha}(AL)^{1-\alpha}, \tag{1}$$

where $0 < \alpha < 1$. The labour force is constant. Gross investment in capital equals a fraction s of total output and capital does not depreciate. The stock of knowledge A accumulated as a by-production of output production (learning by doing) according to the equation

$$\dot{A} = BY. \tag{2}$$

Derive the steady state rates of growth of A, K and Y. What is the impact of an increase in s and B on them?

2. Consider a consumer living for two periods. Her utility function is

$$\log(c_1) + \beta \log(c_2) \tag{3}$$

where c_1 and c_2 are respectively consumption in the first and second period and $0 < \beta < 1$ is the factor at which she discounts future utility. The consumer is born with no assets, receives labour income y_1 and y_2 respectively in period 1 and 2 and can freely borrow and lend at the market interest rate r.

   (a) Write down the consumer dynamic budget constraint.

   (b) Impose solvency and derive the consumer intertemporal budget constraint.

   (c) Write down the Euler equation. What does it imply about the consumer consumption profile?

   (d) Use the Euler equation, together with the intertemporal budget constraint to solve for consumption and saving in period 1.