Macroeconomics A

Solution to problem set 2

1. Assuming K depreciates at rate δ (this information was missing in the text of the problem set!) the rate of growth of K can be obtained by from the capital accumulation equation and it is

$$\frac{\dot{K}}{K} = sK^{\alpha-1}T^{\beta}(AL)^{1-\alpha-\beta} - \delta. \tag{1}$$

   - In steady state $\dot{K}/K$ has to be constant which requires the average product of capital $K^{\alpha-1}T^{\beta}(AL)^{1-\alpha-\beta}$ to be constant or

$$(\alpha-1)\frac{\dot{K}}{K} + \beta\frac{\dot{T}}{T} + (1-\alpha-\beta)\left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right) = 0, \tag{2}$$

     or

$$\frac{\dot{K}}{K} = \frac{1-\alpha-\beta}{1-\alpha}(g+n). \tag{3}$$

     Since in steady state Y/K is constant, the above is also the steady state rate of growth of output.

   - The rate of growth of output per worker is

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - n = \frac{1-\alpha-\beta}{1-\alpha}(g+n) - n. \tag{4}$$

     which is positive if

$$(1-\alpha-\beta)g > \beta n. \tag{5}$$

     Intuition:

     (a) If $\beta = 0$ we are back to Solow model and $g > 0$ is necessary for the model to have IRS to all non-fixed factors (i.e. A has to be non-fixed).

     (b) If $\beta > 0$ the model has DRS to K, L and land is fixed. Unless A grows the model has DRS to the non-fixed factors and growth eventually dies out (Malthusian growth).

2. The accumulation equation for K is

$$\frac{\dot{K}}{K} = s_K K^{\alpha-1}H^{\beta}(AL)^{1-\alpha-\beta} - \delta, \tag{6}$$

   and the corresponding equation for human capital H is

$$\frac{\dot{H}}{H} = s_H K^{\alpha}H^{\beta-1}(AL)^{1-\alpha-\beta} - \delta. \tag{7}$$

(a) The economic equilibrium is a vector of functions of time $[K_t, H_t, Y_t]$ such that (6), (7) and the production function hold.

(b) The accumulation equation in efficiency units of labour are

$$\frac{\dot{\tilde{k}}}{\tilde{k}} = s_K \tilde{k}^{\alpha-1}\tilde{h}^{\beta} - (\delta + g + n), \tag{8}$$

and

$$\frac{\dot{\tilde{h}}}{\tilde{h}} = s_H \tilde{k}^{\alpha}\tilde{h}^{\beta-1} - (\delta + g + n). \tag{9}$$

For a steady state equilibrium to exist the average product of physical and human capital have to be constant. This holds only if $\tilde{k}$ and $\tilde{h}$ do not grow.

(c) Since the average product of capital is constant in steady state, $\tilde{y}$ must grow at the same rate as $\tilde{k}$; i.e. zero. This implies that output per capita grows at the rate of technological progress g.

Same predictions as Solow growth model. The intuitions is that the model has the same properties: DRS to K, H, L. Thus unless A grows the model has DRS to non-fixed factors.

(d) Imposing $\dot{\tilde{k}}/\tilde{k} = \dot{\tilde{h}}/\tilde{h} = 0$ in (8) and (9) yields

$$s_K \tilde{k}^{\alpha-1}\tilde{h}^{\beta} = s_H \tilde{k}^{\alpha}\tilde{h}^{\beta-1} \tag{10}$$

or

$$\frac{\tilde{h}}{\tilde{k}} = \frac{s_H}{s_K}. \tag{11}$$

Intuition: in steady state the effective rate of depreciation is the same for both K and H. Therefore the gross investment rate must be the same which requires a higher marginal product for the capital into which a smaller proportion of output is invested.