Macroeconomics A

Solution to problem set 3

1. Since capital does not depreciate the capital accumulation equation is

$$\frac{\dot{K}}{K} = sK^{\alpha-1}(A(1-\alpha_L)L)^{1-\alpha} = s\left(\frac{K}{AL}\right)^{\alpha-1}(1-\alpha_L)^{1-\alpha}. \tag{1}$$

Dividing both sides of equation (2) in the text of the exercise one can also write

$$\frac{\dot{A}}{A} = \delta(\alpha_L L)^\lambda A^{\varphi-1}. \tag{2}$$

In steady state variables need to grow at a constant rate. For this to be the case for the capital stock, it follows from equation (1) that K/AL has to be constant which implies

$$\frac{\dot{K}}{K} = \frac{\dot{A}}{A} + \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + n. \tag{3}$$

Constant growth also requires the RHS of equation (2) to be constant or

$$\lambda\frac{\dot{L}}{L} + (\varphi-1)\frac{\dot{A}}{A} = 0, \tag{4}$$

or

$$\frac{\dot{A}}{A} = \frac{\lambda n}{1-\varphi}. \tag{5}$$

(a) This is the derivative with respect to L of the RHS of equation (2) in the text of the exercise; i.e.

$$\delta\lambda(\alpha_L L)^{\lambda-1}A^\varphi.$$

Since $\lambda < 1$, it is decreasing in L, which suggests congestion from "stepping on toes."

(b) If $\varphi < 1$ equation (5) is determinate and given the rate of growth of A. From (3), K/L grows at the same rate as A. This is also the rate of growth of Y/L, since Y/K is constant in steady state. The rates tend to infinity as $\varphi \to 1$ as the marginal product of knowledge tends to a constant.

(c) In this case, equation (5) is indeterminate, but equation (2) implies

$$\frac{\dot{A}}{A} = \delta(\alpha_L L)^\lambda. \tag{6}$$

This is also the rate of growth of K/L and Y/L.

(d) No effect under case (b) and the rate of growth of A is increased in case (c).

In case (b) the stock of knowledge grows temporarily faster and the stock of capital temporarily more slowly as the increase in α_L means workers move from producing output (and capital) to producing ideas. Since marginal returns to both reproducible factors are decreasing (weak scale effects) there is no long run effect on growth rates.

In case (c) the impact on the rate of growth of A is permanent. The capital stock rate of growth first falls, but eventually increases as it approaches its new, higher, steady state value.

(e) The production function displays CRS to K and L or K and A. So it displays increasing returns only to *all factors which are not fixed in steady state* only if <u>all</u> the free factors grow in steady state. This is true in case (b) where all three factors grow. In case (c) L is constant, so the production function for output displays CRS to non-fixed factors and steady state growth is driven by constant marginal returns to knowledge accumulation.