Macroeconomics A

Solution to problem set 9

1. Consider the following problem. The aggregate supply equation is given by

$$y_t = (a + e_t)(\pi_t - \pi_t^e), \tag{1}$$

where y is output, π is actual inflation and π^e the private sector's expected inflation; a is a positive constant and e an independent and non-autocorrelated random shock with zero mean and variance σ^2. The monetary authority sets the rate of inflation after observing the shock e. Private sector expectations are rational and formed before the authorities determine actual inflation. Private agents never observe e. The policymaker welfare function is given by

$$W = \lambda y - \frac{\pi^2}{2}, \tag{2}$$

with $\lambda > 0$.

(a) Under discretion the policy maker maximizes W subject to the aggregate supply taking agents' expectations as given; i.e. she solves

$$\max_{\pi_t} \lambda[(a + e_t)(\pi_t - \pi_t^e)] - \frac{\pi_t^2}{2} \tag{3}$$

The FOC is

$$\pi_t = \lambda(a + e_t). \tag{4}$$

which implies $\pi_t^e = \lambda a$ and $y_t = \lambda(a + e_t)e_t$ and

$$W^d = \lambda^2(a + e_t)e_t - \frac{[\lambda(a + e_t)]^2}{2} \tag{5}$$

and

$$EW^d = \frac{\lambda^2\sigma^2 - (\lambda a)^2}{2}. \tag{6}$$

(b) A credible commitment to the rule implies $\pi^e = c$ and $y_t = (a + e_t)de_t$. It follows that it is

$$W^c = \lambda(a + e_t)de_t - \frac{[(c + de_t)]^2}{2} \tag{7}$$

and

$$EW^c = \frac{2\lambda d\sigma^2 - (c^2 + d^2\sigma^2)}{2}. \tag{8}$$

Which is maximized for $c = 0$ and $d = \lambda$. That is at an optimum it is $EW^c = (\lambda^2\sigma^2)/2$. The commitment might not be credible.

(c) Inflation will now be chosen by the central banker to maximize

$$\max_{\pi} \mu[(a+e_t)(\pi_t - \pi_t^e)] - \frac{\pi_t^2}{2}. \tag{9}$$

The FOC is

$$\pi_t = \mu(a+e_t). \tag{10}$$

which implies $\pi_t^e = \mu a$ and $y_t = \mu(a+e_t)e_t$ and

$$W^{cb} = \lambda\mu(a+e_t)e_t - \frac{[\mu(a+e_t)]^2}{2}. \tag{11}$$

Note that the above is **government** welfare.
It follows that

$$EW^{cb} = \lambda\mu\sigma^2 - \frac{\mu^2[a^2+\sigma^2)]}{2}. \tag{12}$$

This is minimized for $\mu = \frac{\lambda\sigma^2}{a^2+\sigma^2}$.
The government cares about the shock because it affects the *level* of income given that it affects both the slope of the aggregate supply and $\pi - \pi^e$. At an optimum it is

$$EW^{cb} = \frac{\lambda^2\sigma^4}{2(\alpha^2+\sigma^2)}. \tag{13}$$

The ratio between expected welfare here and under commitment is

$$\frac{EW^{cb}}{EW^c} = \frac{\sigma^2}{\alpha^2+\sigma^2} < 1, \tag{14}$$

as $a > 0$. Government welfare is higher under commitment.