

## Macroeconomics B

### Problem set 5

*This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.*

1. Consider the overlapping generation model with production studied in the lectures. Assume that there is no growth in total factor productivity but that the population grows at the exogenous rate  $n$ . The government introduces a pay-as-you-go social security system with the following characteristics. Social security imposes a tax  $\tau$  on young people, so that their disposable income is  $W_t - \tau$ . The tax is used to pay a pension  $\tau(1+n)$  to old people in the same time period. (Note: there are  $(1+n)$  young agents for each old agent). Compare the steady state of this economy to the steady state of the economy studied in the lectures when  $\tau = 0$ . In particular compare individual consumption, individual saving and aggregate saving in the two economies.
2. Consider an overlapping generations economy with constant population  $L$  and no capital depreciation. Assume that every agent born at time  $t$  has the utility function

$$u(c_t^1, c_{t+1}^2) = \log c_t^1 + (1 + \rho)^{-1} \log c_{t+1}^2.$$

Agents' gross labour income equals their wage when young and is zero when old. Firms are price takers on both the good and factor markets. They produce output using capital and labour according to the production function  $Y_t = K_t^\alpha L_t^{1-\alpha}$  with  $0 < \alpha < 1$ . Capital does not depreciate. The government taxes only labour income at a proportional rate  $1 > \tau > 0$  and uses the revenue to finance wasteful public expenditure. All agents have perfect foresight.

- (a) Determine the aggregate saving function. Derive the difference equation which relates the stock of capital per head  $k_{t+1}$  to its past value  $k_t$ . Sketch  $k_{t+1}$  as a function of  $k_t$ . What is the sign of the aggregate saving ratio in steady state and outside the steady state?
- (b) Suppose the economy is initially in steady state. The government unexpectedly increases the tax rate  $\tau$  and is expected to keep it at its new higher level forever after. Derive the dynamics of the stock of capital per head.
- (c) Determine whether for  $\tau = 0$  the economy is dynamically inefficient or not (hint: i.e. whether its capital stock exceeds the golden rule level or not). If the decentralized equilibrium with  $\tau = 0$  is dynamic inefficient, determine the value of  $\tau$  which ensures the decentralized equilibrium is Pareto optimal.