PhD Macro Section

Exam paper

Solve the following problem on the computer using whatever programming language you fancy. You will be graded on the basis of an individual oral presentation with us on Friday 10 December. Please print out your code and whatever other output the problem requests and bring it with you on the day.

Consider the following consumer problem[1]

$$\begin{aligned} &\max_{c_t, a_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{c_t^{1-\gamma} - 1}{1-\gamma} \\ &\text{s.t } a_{t+1} = (1+r)a_t + 1.5e^{y_t} - c_t \\ &\quad a_{t+1} \geq 0 \\ &\quad a_0, z_0 \text{ given.} \end{aligned}$$

The term $1.5e^{y_t}$ is the agent labour's income at time t where the shock y_t is normally distributed with mean zero and variance σ^2.

The purpose of this problem set is to solve for the policy functions using discretized value function iteration and the endogenous grid method.

The parameter values are:
$\beta = 0.96,\ r = 0.02,\ \gamma = 3,\ \sigma = 0.2.$

1. Write the consumer problem in recursive form. Since the problem is stationary we write $c,\ a,\ y$ for $c_t,\ a_t,\ y_t$ and $a',\ y'$ for $a_{t+1},\ y_{t+1}$ in what follows. In choosing your state vector, use (a, y) and not, as in Aiyagari, $z = a(1+r) + 1.5e^y$.
2. Construct a grid of 7 Gaussian-Hermite quadrature nodes for the shock y.
3. Construct a grid of 100 equidistant nodes for a.
4. (a) Use discretized value function iteration to solve for the policy functions for current consumption c and end-of-period financial wealth a'.

   (b) For your chosen state vector (a, y):
   - Plot the saving function $a'(a, y)$ against a for y equal to its highest value on the grid. Verify that your chosen upper bound for the grid a is large enough. If not increase it, construct a new grid for a and go back to (a).
   - Evaluate accuracy in the following way. For each pair (a, y) :
     (i) construct the function

$$g(a, y) = \beta(1+r)E\{[1.5e^{y'} + a'(a, y)(1+r) - a'(a'(a, y), y')]^{-\gamma}\}; \quad (1)$$

[1] The problem is a partial equilibrium version of Aiyagari's (1994) paper, Uninsured Idiosyncratic Risk and Aggregate Saving, *The Quarterly Journal of Economics,* 109, p 659-684. A similar version is in Chapter 4.1 in Ljungqvist and Sargent.

Note that $a'(a'(a,y),y')$ denotes the choice 2 periods ahead since the function $a'(.)$ is evaluated at $a'(a,y)$ and y'.
(ii) compute the consumption level

$$\hat{c}(a,y) = g(a,y)^{-1/\gamma} \tag{2}$$

which satisfies the Euler equation;
(iii) report the maximum over all (a,y) of the log Euler error

$$\log_{10} |\hat{c}(a,y)/c(a,y) - 1|, \tag{3}$$

where $c(a,y)$ is the consumption function computed in point (a).

5. Redo point 4. using the endogenous grid method instead of value function iteration. Compare the maximum log Euler error in the two cases.