LECTURE 2

1 What generates growth?

Why does growth in income per capita eventually stop without exogenous technological progress?

Output per capita can be written as

$$y = \frac{Y}{L} = \frac{Y}{K}\frac{K}{L} = \frac{Y}{K}k. \tag{1}$$

It grows only if:

1. the average product of capital grows or
2. capital per worker grows (or both).

The capital accumulation equation

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta \tag{2}$$

implies that in steady state equilibrium Y/K has to be constant.

Sustained (i.e. steady state) growth in per capita income y requires the stock of capital per worker k to grow.

In fact, $\dot{y}/y = \dot{k}/k$ in steady state.

Consider an economy with:

- $Y = K^{\alpha} L^{\beta}, \quad \alpha, \beta > 0.$
- No exogenous technological progress.

The capital accumulation equation is:

$$\frac{\dot{K}}{K} = s\frac{Y}{K} - \delta = sK^{\alpha-1}L^{\beta} - \delta. \tag{3}$$

If a steady state equilibrium exists Y/K has to grow at zero rate or

$$(\alpha - 1)\frac{\dot{K}}{K} + \beta\frac{\dot{L}}{L} = 0 \tag{4}$$

or

$$\frac{\dot{K}}{K} = \frac{\beta}{1-\alpha}n. \tag{5}$$

Steady state growth in y requires

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = \frac{\beta}{1-\alpha}n - n = \frac{\beta+\alpha-1}{1-\alpha}n > 0. \tag{6}$$

This is the case only in two cases:

1. $\beta + \alpha > 1$ and $n > 0$. **Increasing returns to all non-fixed factors and at least one is exogenously increasing.**
2. $\alpha = 1$ and $n = 0$. **Constant returns to capital and no growth in complementary factors.**

   This case is slightly less obvious since in equation (6) we have an indeterminate quantity 0/0. The capital accumulation equation (3) can be rewritten as

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - n = sk^{\alpha-1}L^{\beta+\alpha-1} - \delta - n. \tag{7}$$

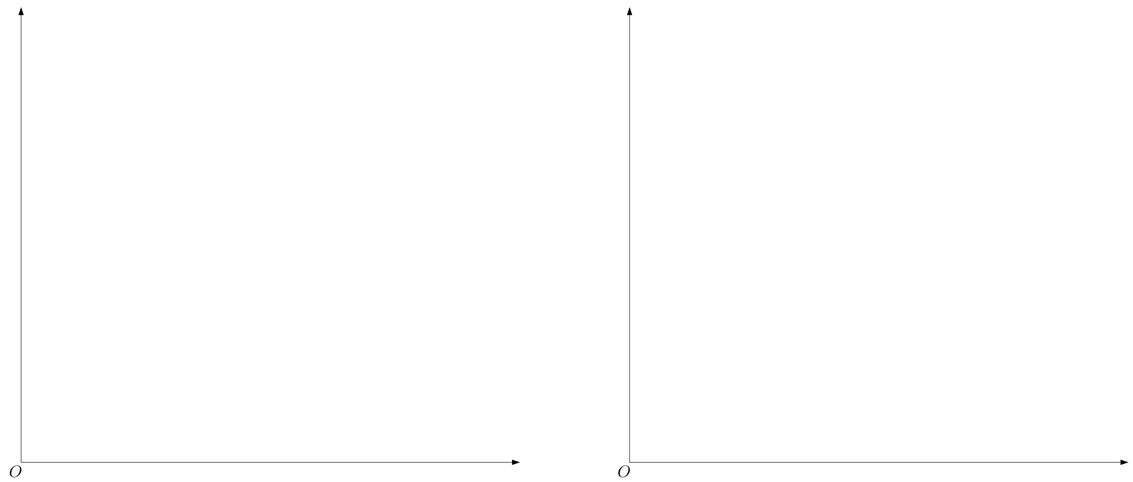
   If $\alpha = 1$ it becomes

$$\frac{\dot{k}}{k} = sL^{\beta} - \delta - n. \tag{8}$$

Definition 1 *A* reproducible *factor is one which is endogenously accumulated.*

Example: In the Solow growth model, capital is the only reproducible factor.

As long as the average product of the reproducible factor declines, there comes a point where its investment rate equals its depreciation rate and accumulation stops. Increase in the labour force pulls the average product up (because of complementarity) but *with CRS to the reproducible factor and labour* only at the same rate as population growth.



$\alpha < 1$ $\alpha = 1$

Without either (1) IRS to all non-fixed factors or (2) constant marginal product to the reproducible factor no long run growth in per capita variables.

1. **Increasing returns to all non-fixed factors and at least one is exogenously increasing.** In steady state

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = \frac{\beta + \alpha - 1}{1 - \alpha} n > 0. \tag{9}$$

   Steady state growth rate in y independent of size of economy (i.e. L) and policy parameters (e.g. s). $\rightarrow$ Weak scale effects.

   IRS to K and L, while useful as illustration, do not make sense. Replicating a factory by a factor λ we scale the output by more than λ!

   DRS are usually justified with congestion, IRS require spillovers. If IRS are present they must be due to some non-rival input $\rightarrow$ ideas, knowledge.

   The Solow growth model belongs here. IRS to A, L, K together, though CRS to K, L. A is *non-rival* and *non-excludable.*

2. **Constant returns to capital and no growth in complementary factors.**

$$\frac{\dot{y}}{y} = \frac{\dot{k}}{k} = sL^{\beta} - \delta - n. \tag{10}$$

Steady state growth rate in y depends on size of economy policy parameters.
$\rightarrow$ <u>Strong scale effects</u>.

2 Endogenous growth

Neoclassical growth theory does not explain:

- Determinants of long run growth
- Determinants of long run differences in growth rates
- Policy does not affect long run growth.

This is where endogenous growth comes in.

2.1 The basic intuition: the AK model

The AK model (Rebelo 1991) emphasizes in a very powerful way the importance of constant marginal product to reproducible factors.

- The main novelty is that the production function now takes the form

$$Y = AKL. \tag{11}$$

  The production function has constant marginal (and average) return AL to the (unique) reproducible factor K and increasing returns to K and L.

- The rates of growth of A and L are zero or

$$\frac{\dot{A}}{A} = \frac{\dot{L}}{L} = 0. \tag{12}$$

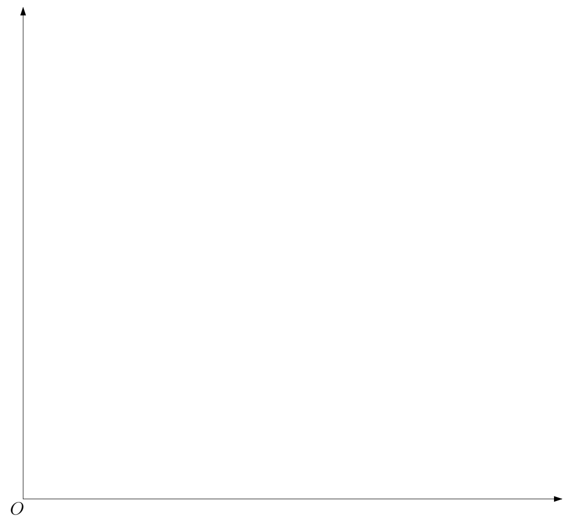
The accumulation equation is now

$$\dot{K} = sY - \delta K = sAKL - \delta K. \tag{13}$$

Definition 2 *The steady state equilibrium is a vector of functions* $\{K_t, Y_t\}$ *in which equations* (11) *and* (14) *hold and all variables grow at a constant rate.*

Divide both sides of equation (13) by K to obtain

$$\frac{\dot{K}}{K} = sAL - \delta. \tag{14}$$



Capital accumulation in the AK model.

$Y/K = AL$ is constant. So both capital and income grow at the same rate $sAL - \delta$.

Three important points:

- The rate of growth of variables depends on country-specific policy parameters (s, δ).
- The model has *strong scale effects*: the steady state rate of growth depends on the size of the labour force.
- No transitional dynamics. The steady state equilibrium is the unique equilibrium.

2.2 Externalities: learning by doing

The model is due to Arrow (1962). Historically, the first one to emphasize accumulation of non-rival knowledge.

- Standard CRS production function with decreasing returns to capital. Cobb Douglas:

$$Y = K^{\alpha}(AL)^{1-\alpha}. \tag{15}$$

- The stock of knowledge A is positively related to the stock of capital K.

$$A = BK^{\phi}. \tag{16}$$

  Intuition: learning by doing (on the job). Task complexity and instructiveness positively related to capital stock.

- The effect of K on A is an externality as A is non-rival (available to all firms in the economy).

Replacing for A in Y, we can write the capital accumulation equation

$$\frac{\dot{K}}{K} = sK^{\alpha-1}\left(BK^{\phi}L\right)^{1-\alpha} - \delta = \frac{\dot{K}}{K} = sK^{(\phi-1)(1-\alpha)}(BL)^{1-\alpha} - \delta. \tag{17}$$

Definition 3 *The steady state equilibrium is a vector of functions* $\{K_t, Y_t, A_t\}$ *in which equations* (15), (16) *and* (17) *hold and all variables grow at a constant rate.*

For $\dot{K}/K$ to be constant the term $K^{(\phi-1)(1-\alpha)}(BL)^{1-\alpha}$ has to be constant (grow at zero rate).

Take logs and time derivatives of it to obtain

$$(\phi-1)(1-\alpha)\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L} = 0. \tag{18}$$

Three cases to consider:

1. $\phi > 1$. Explosive growth for any $n \geq 0$. No steady state.

2. $\phi = 1$. Constant marginal return to K. Equation (17) reduces to $\dot{K}/K = s(BL)^{1-\alpha} - \delta$. Effectively the same as AK model. Policy matters and *strong* scale effects $\rightarrow$ Explosive growth if $n > 0$.

3. $\phi < 1$. DRS to A and K together. Rearrange equation (18) to obtain

$$\frac{\dot{K}}{K} = \frac{n}{1-\phi}. \tag{19}$$

Steady state growth only if $n > 0$. *Weak scale effects*: steady state rate of growth depends on *rate* of population growth.

- Case 2. is appealing as it can potentially explain persistent cross-country differences in rates of growth on the basis of differences in policy parameters (assuming A is excludable and different across countries). Consistent with positive correlation between growth and saving rates. Yet, the model would also predict that the rate of growth in one country should change with changes in e.g. saving rate. **Counterfactual**.

- Case 3. would predict instead that countries with higher population growth should grow faster. This is clearly not the case (e.g. Africa). The model makes sense as a model of growth in the *world* stock of knowledge. In fact, it is consistent with the pattern of growth rates over centuries.

The Arrow model highlights the two important issues of scale and policy effects which are common to all the endogenous growth literature. Whichever stance one takes the endogenous growth literature is unlikely to be able to explain cross country differences in growth rates without introducing barriers to technological adoption.

2.3 Research and development

The model is due to Romer (1990). The accumulation of knowledge is no longer a byproduct of capital accumulation, but it is the outcome of endogenous investment (R&D). It depends, among other things, on the number of people engaged in R&D.

- Two sectors: output sector and R&D sector. α_L, α_K are respectively the shares of total employment and total capital used in the R&D sector.
- Production function for output
$$Y = [(1-\alpha_K)K]^{\alpha}[A(1-\alpha_L)L]^{1-\alpha}. \tag{20}$$
- R&D accumulation
$$\dot{A} = \overline{\lambda}\alpha_L L. \tag{21}$$
Each researcher has a probability $\overline{\lambda}$ of discovering a new idea in each time period.
- While agents take λ as given, in reality it is
$$\overline{\lambda} = BA^{\theta}(\alpha_K K)^{\beta}(\alpha_L L)^{\gamma-1}, \tag{22}$$

with $\beta, \gamma \geq 0$ but θ unrestricted. If $\gamma < 1$ "stepping on toes" (congestion in R&D sector). If $\theta > 0$ "standing on shoulders of giants". Since agents take λ as given they do not internalize the externalities in the choice of number of researcher and R&D investment.

- The capital accumulation equation is the usual one

$$\frac{\dot{K}}{K} = s\frac{[(1-\alpha_K)K]^{\alpha}[A(1-\alpha_L)L]^{1-\alpha}}{K} - \delta. \tag{23}$$

Definition 4 *The steady state equilibrium is a vector of functions* $\{K_t, Y_t, A_t, \bar{\lambda}_t\}$ *in which equations* (20), (21), (22) *and* (23) *hold and all variables grow at a constant rate.*

Replacing for $\overline{\lambda}$ in (21) we can rewrite the knowledge accumulation equation as

$$\dot{A} = BA^{\theta}(\alpha_K K)^{\beta}(\alpha_L L)^{\gamma}, \tag{24}$$

or, dividing both sides by A, as

$$\frac{\dot{A}}{A} = BA^{\theta-1}(\alpha_K K)^{\beta}(\alpha_L L)^{\gamma}, \tag{25}$$

Solving for steady state equilibrium.

For $\dot{A}/A$ to be constant $BA^{\theta-1}(\alpha_K K)^{\beta}(\alpha_L L)^{\gamma}$ has to be constant. For $\dot{K}/K$ to be constant, $[(1-\alpha_K)K]^{\alpha-1}[A(1-\alpha_L)L]^{1-\alpha}$ has to be constant. Therefore the rate of growth of both these expressions has to be zero.

Taking logs and time derivatives of the second one it has to be

$$\frac{\dot{K}}{K} = \frac{\dot{A}}{A} + \frac{\dot{L}}{L}. \tag{26}$$

Taking logs and time derivatives of the first one it has to be

$$0 = (\theta-1)\frac{\dot{A}}{A} + \beta\frac{\dot{K}}{K} + \gamma\frac{\dot{L}}{L} = (\theta+\beta-1)\frac{\dot{A}}{A} + (\beta+\gamma)\frac{\dot{L}}{L} \tag{27}$$

Three cases to consider:

1. $\theta + \beta > 1$. Explosive growth for any $n \geq 0$. No steady state.
2. $\theta + \beta = 1$. CRS to A and K together (the two reproducible factors). Explosive growth if $n > 0$. If $n = 0$ note that it is

$$\frac{\dot{A}}{A} = B\left(\alpha_K \frac{K}{A}\right)^{\beta} (\alpha_L L)^{\gamma}. \tag{28}$$

We need to determine K/A. Replace for $\dot{K}/K$ in (23) using (26) to obtain

$$\frac{\dot{A}}{A} = s\left[(1-\alpha_K)\frac{K}{A}\right]^{\alpha-1} [(1-\alpha_L)L]^{1-\alpha} - (\delta + n). \tag{29}$$

Use (28) and (29) to solve for $\dot{A}/A$. Increase in saving rate and L increase steady state growth. Increases in α_L and α_K have ambiguous effects.

Policy matters and strong scale effects (that's why explosive growth if $n > 0$).

3. $\theta + \beta < 1$. DRS to A and K together. Equate the expression (27) to zero and rearrange to obtain

$$\frac{\dot{A}}{A} = \frac{\beta + \gamma}{1 - \theta - \beta} n. \tag{30}$$

Steady state growth only if $n > 0$.

<u>Weak scale effects</u>: steady state rate of growth depends on *rate* of population growth.