LECTURE 8

Nominal rigidities imply that demand shocks have real effects, though,under rational expectations, this is the case only for *unexpected* shocks.

One unpalatable feature of rigid prices/wages though is that they imply sub-optimal behaviour. Agents forego gains from optimal price/wage setting.

We now introduce a model in which the real effect of demand shocks stems not from nominal rigidities but from the inability to distinguish (nominal) aggregate shocks from (real) sector-specific one. The model is due to Lucas (1972) though the original insight is actually due to Phelps.

1 Lucas-Phelps' island model.

- The economy has many sectors (islands). Agents in one sector cannot observe what happens in other sectors.
- All variables are in logs.
- The demand for the good produced in sector i at time t is given by

$$y_t^i = b(p_t^i - E\left[p_t | I_t, p_t^i\right]). \tag{1}$$

Producers expand production if they think the price of their product is above the average price (the demand for their product is relatively high). They respond only to shocks to the relative price of their product (real shocks), but not to shocks that increase all prices by the same amount, leaving $(p_t^i - E\left[p_t | I_t, p_t^i\right])$ unaffected.
Their expectation about the average price is conditional on all information available to them (i.e. past realizations of all variables and the model of the economy + their current own price realization) .

- Individual prices are subject to normally-distributed i.i.d. shocks z_t^i with mean zero and variance σ_z^2. That is

$$p_t^i = p_t + z_t^i. \tag{2}$$

- Before observing their own price realization p_t^i their price expectation is rational conditional on all information excluding p_t^i. Because of rationality, their forecast error ϵ_t is uncorrelated with (cannot be predicted on the basis of) the information set I_t.

$$p_t = E\left[p_t | I_t\right] + \epsilon_t. \tag{3}$$

The variance of ϵ_t is the same as the variance of p_t. We therefore denote it by σ_p^2

- Since p_t and p_t^i are positively correlated agents can use the realization of the price of their own product to improve on their original forecast.

$$E\left[p_t|I_t, p_t^i\right] = E\left[p_t|I_t\right] + E\left[\left(p_t - E\left[p_t|I_t\right]\right) | \left(p_t^i - E\left[p_t^i|I_t\right]\right)\right]. \qquad (4)$$

  One way to think about the second term is as the fitted value of the regression of the part of p_t which cannot be forecast on the basis of I_t on the part of p_t^i which cannot be forecast on the basis of I_t. Alternatively for the estimate of the regression coefficient to be unbiased the regression needs to include an intercept term or be run on deviations from the means (the only important difference is that the means depend on the whole model. They are not necessarily constant). If ϵ_t is normally distributed the fitted value of the regression estimated by OLS coincides with the conditional expectation (i.e. it is not only unbiased but also efficient).

- Notice that $E\left[p_t^i|I_t\right] = E\left[p_t|I_t\right]$ because of (2)

- The fitted value is $\theta\left(p_t^i - E\left[p_t^i|I_t\right]\right)$, with

$$\theta = \frac{E[(p_t - E[p_t|I_t])(p_t^i - E[p_t^i|I_t])|I_t, p_t^i]}{E[(p_t^i - E[p_t^i|I_t])^2|I_t, p_t^i]} = \frac{E[\epsilon_t(\epsilon_t + z_t^i)]}{E[(\epsilon_t + z_t^i)^2]} = \frac{\sigma_p^2}{\sigma_p^2 + \sigma_z^2}. \tag{5}$$

  $\theta \to 0$ if σ_z^2 is large relative to σ_p^2. The difference $p_t^i - p_t$ is highly variables, hence p_t^i little informative about p_t. Vice versa $\theta \to 1$ if σ_z^2 is small relative to σ_p^2. In such a case idiosyncratic shocks are negligible and p_t^i is highly informative about p_t.

- Using $E\left[p_t^i|I_t\right] = E\left[p_t|I_t\right]$ we can then write

$$E\left[p_t|I_t, p_t^i\right] = E\left[p_t|I_t\right] + \theta(p_t^i - E[p_t|I_t]). \tag{6}$$

  Replacing in (1) we obtain

$$y_t^i = b(1-\theta)(p_t^i - E[p_t|I_t]). \tag{7}$$

  Supply curve is closest to vertical the closest θ is to 1, as producers do not respond to shocks as they are mostly aggregate shocks. Average over the i goods we obtain the aggregate supply curve

$$y_t = b(1-\theta)(p_t - E[p_t|I_t]). \tag{8}$$

which implies $E(y_t|I_t) = 0$.

- Notice that θ is still unknown as it depends on the yet unknown and endogenous variance of the aggregate price level σ_p^2.
- Aggregate demand

$$m_t = p_t + y_t - v_t. \tag{9}$$

Which implies

$$p_t - E(p_t|I_t) = m_t - E(m_t) - y_t - [v_t - E(v_t)] \tag{10}$$

- Replace for y_t using (7) to obtain

$$[1 + b(1-\theta)]\,[p_t - E(p_t|I_t)] = m_t - E(m_t) - [v_t - E(v_t)]. \tag{11}$$

Assume shocks to m_t and v_t are uncorrelated and normally distributed. Taking squares and expectations this implies

$$\sigma_p^2 = \frac{\sigma_m^2 + \sigma_v^2}{[1 + b(1-\theta)]^2} \tag{12}$$

which gives σ_p^2 as a function of the exogenous variances σ_m^2 and σ_v^2.

2 Only unanticipated policy matters

Rational expectations imply that only shocks which cannot be forecast on the basis of the agents' information set matter.

To see this note that for both the sticky wage and the Lucas model the equilibrium of the system is the vector $[y_t, p_t, Ep_t]$ satisfying the aggregate demand curve

$$m_t + v_t = y_t + p_t, \tag{13}$$

the aggregate supply curve

$$y_t = \gamma(p_t - Ep_t) \tag{14}$$

and where Ep_t is formed rationally; i.e. using the system of the two equations and all other relevant information. Therefore it satisfies

$$Ep_t = Em_t + Ev_t. \tag{15}$$

Anticipated shocks to monetary policy change m_t and Em_t by the same amount.

- Graphical intuition. Since Ep_t and Em_t change by the same amount as m_t both AD and AS shift up by the same amount and output is unaffected.

- Algebraic solution. Subtract (13) from its expectation to obtain

$$p_t - Ep_t = (m_t - Em_t) + (v - Ev_t) - y_t. \tag{16}$$

  Replace in (14) and solve for output as a function of exogenous variables to obtain

$$y_t = \frac{\gamma}{1+\gamma}\left[(m_t - Em_t) + (v - Ev_t)\right]. \tag{17}$$

Only unexpected changes in exogenous variables affect equilibrium output. In the absence of unexpected shocks the (log) of output is at its perfect information equilibrium of zero.

3 Ineffectiveness of systematic policy

A monetary policy rule is a function mapping past and present realizations of variables to a value for the nominal money supply m_t. An example is the constant money growth rule

$$m_t = c + m_{t-1} + u_t. \tag{18}$$

The (log) of the money supply equals its past value plus a constant increases plus a random shock u_t with zero mean.

- The shock u_t is called the *unsystematic* (random) component of the monetary policy rule, as it is not related to observable variables.
- $c + m_{t-1}$ is the *systematic* part of the monetary policy rule, as it is related in a systematic way to observable variables.

Lucas-Sargent-Wallace Policy Ineffectiveness Proposition. If

1. agents have rational expectations;
2. there are no nominal rigidities;
3. the policymaker has no informational advantage over private agents

then the systematic component of monetary policy cannot affect the average level nor the variance of output.

The unsystematic component cannot affect the level and should be set to zero to minimize output variability.

To see this note that equilibrium output satisfies

$$y_t = \frac{\gamma}{1+\gamma}\left[(m_t - Em_t) + (v_t - Ev_t)\right]. \tag{19}$$

If monetary policy is conducted according to the rule (18) it is $(m_t - Em_t) = u_t$, as the rule and m_{t-1} belong to the information set of private agents. Replacing in (19) we obtain

$$y_t = \frac{\gamma}{1+\gamma}\left[u_t + (v_t - Ev_t)\right]. \tag{20}$$

Since u_t has mean zero by construction it cannot affect expected output. Assuming u_t and v_t are uncorrelated the variance of output is given by

$$\sigma_y^2 = \left(\frac{\gamma}{1+\gamma}\right)^2 [\sigma_u^2 + \sigma_v^2]. \tag{21}$$

The best the policymaker can do is to run policy so as to minimize σ_y^2, by setting the variance of the shock it controls (u_t) to zero. In other words the best the policymaker can do is not to randomize its policy. The PIP fails though if one of the three assumptions on which it is based does not hold.

E.g. Policymaker has informational advantage over private agents. It observes the current aggregate demand shocks v_t. Assume v_t has zero mean. Consider the systematic monetary policy rule

$$m_t = c + \delta v_t. \tag{22}$$

Since v_t does not enter the information set of private agents it is $Em_t = c$ and $m_t - Em_t = \delta v_t$. Therefore it is

$$y_t = \frac{\gamma}{1+\gamma}\left[\delta v_t + v_t\right]. \tag{23}$$

The policymaker can fully stabilize output by setting $\delta = -1$ by fully offsetting money demand shocks by cutting the money supply by an equal amount. Yet, if the reason for policy ineffectiveness is that the policymaker has got better information a better course of action would be to publish the information. By doing so the economy would still be fully stabilized as private agents would adjust their expectations. Furthermore, information about aggregate variables is readily available.

For systematic policy to have any role under rational expectations their must exist nominal rigidities (e.g. nominal wages predetermined at the beginning of

the period) that do not allow private agents to adapt their behaviour *within the period* in response to new information.

If policymakers do not have an informational advantage, though, price rigidities are not enough for systematic policy to be effective. Since the policymaker cannot respond to innovations to shocks (does not observe them in the same way that private agents do not), shocks must be serially correlated so that policymakers can usefully respond to their systematic component. We will see this next week.

4 Empirical tests

The class of models considered above imply that the unsystematic component of monetary policy is effective. One might be tempted to test for this result by running a regression of this kind

$$y_t = \alpha_0 + \alpha_1 m_t + \epsilon_t \tag{24}$$

There are two types of problems in running the above regression

- Endogeneity bias if m_t is endogenous. This is most easily seen in the case we

have considered in which the policymaker can fully stabilize demand shocks because of an informational advantage. Exactly because the policymaker is successful, output is always at its fully employment level and is uncorrelated with the money supply. This is the usual problem with endogenous regressors which are correlated with the error term.

- The estimated equation may not be stable to changes in the policy rule as its parameters depend on the policy rule through expectations. This result goes under the name of **Lucas critique**. Assume monetary policy is indeed exogenous and governed by the rule

$$m_t = c + \epsilon_t. \tag{25}$$

If velocity shocks have zero mean, the true equation for equilibrium output is

$$y_t = \frac{\gamma}{1+\gamma}\left[(m_t - Em_t) + v_t\right] = -\frac{\gamma}{1+\gamma}c + \frac{\gamma}{1+\gamma}m_t + \frac{\gamma}{1+\gamma}v_t \tag{26}$$

and the econometrician will find $\alpha_1 = \gamma/(1+\gamma)$. Yet, the econometrician would be wrong in concluding from his/her result that a change in the

systematic component of monetary policy c would affect output. In fact, the intercept of the equation is not stable and changes with c. In other words, the expected price level is not invariant to changes in the policy rule and the AS shifts up with changes in the policy rule. One cannot use reduced form equations to make inference about the effect of monetary policy (or policy variables in general) since expectations are *endogenous* and enter the parameters of the estimated equations. One need to estimated jointly expectations and behavioural equations. Alternatively, one needs to be able to identify the *unpredictable* component of monetary policy.

Testing the Lucas-Phelps model. Provided one is able to identify the exogenous and unpredictable change in the money supply the model predicts a smaller impact of the innovation in monetary policy in economies in which the variance of the aggregate price level is higher. This prediction is supported by the data.

5 Logical problems with the Lucas imperfect information model

- The model is driven by supply responses, so we need large labour supply elasticities to get a plausible response. Estimated labour supply elasticity are small.
- The model implies that the effect of monetary policy is not persistent.
- Information about aggregate variables is readily available. For the mechanism to be relevant one needs to have costs of processing the information.