LECTURE 10

1 Time-inconsistency of optimal (monetary) policy

Up to now we have assumed that monetary policy rules were *exogenous* mappings from economic variables to the money supply. We assumed that once the rule was chosen policymakers were fully committed to, and could not deviate from, it.

We now relax this assumption. This is important for various reasons.

1. In practice, mandates for central banks may be very loose (e.g. US Federal Reserve) and in the past monetary policy was run by the government without any explicit rule (e.g. UK before 1997). Hence, deviations from some (possibly implicit) rule are likely.

2. In such a case the objectives of the policymaker in charge of monetary policy determine incentives to deviate.

3. It provides a welfare criterion to evaluate tradeoffs between alternative objectives (e.g. inflation versus output stabilization in response to supply shocks).

In what follows, we assume monetary policy is chosen by the policymaker to maximize its own objective function. Agents are so rational that they not only know the correct model of the economy, but also the policymaker's objective function.

According to this theory the existence of inflation in the long run is due only to the fact that agents *expect* inflation to be non-negative.

- **Positive theory of inflation:** From a positive point of view, the observation that the long run rate of inflation is positive and significant (its average in OECD countries over the past 20 years is about 6%) calls for an explanation. With money being roughly neutral in the long run, it is difficult to understand why policymakers should print money at a rate which results in positive inflation.
- **Normative theory of optimal design of monetary institutions:** The theory has implications for the optimal design of mandates for monetary authorities.

The theory relies on the notion of **time inconsistency**.

2 Time inconsistency

Time inconsistency means that in a dynamic situation agents *ex ante* and *ex post* incentives may differ. More specifically an sequence of values for the choice variable that was optimal as of time t may no longer be so once time $t+1$ arrives, even if no unexpected event has taken place between t and $t+1$.

An example may help. Taxation of capital goods reduces the incentive to invest. So, the government has an incentive to promise that capital will be exempted from tax. Yet, once private agents have invested, the government has an incentive to renege on its earlier pledge, since the investment decision has already taken place and the tax does not distort investment choices any more. If private agents are rational (i.e. understand the government future incentives) they will not believe the government promise and underinvest.

This example highlights two aspects that are crucial for this theory: (a) the rationality of expectations about the future and (b) credibility.

Rational agents will believe the government to take only those actions which are time-consistent, i.e. optimal from the policymaker's *ex post* standpoint.

2.1 Economic environment

Aggregate supply

$$y_t = y_n + \alpha(p_t - p_t^e) + e_t \tag{1}$$

where p_t^e denotes the expectation of the price level p_t at the beginning of period t and y_n is the *natural* output rate which would prevail in the absence of nominal rigidities (including $p_t = p_t^e$). e_t is a white noise shock with zero mean and variance σ_e^2.

We can add and subtract p_{t-1} from the parenthesis in equation (1) and obtain

$$y_t = y_n + \alpha(\pi_t - \pi_t^e) + e_t, \tag{2}$$

where the rate of inflation $\pi_t = p_t - p_{t-1}$ and its expectation at the beginning of time t is $\pi_t^e = p_t^e - p_{t-1}$.

Aggregate demand

$$m = p + y_n, \tag{3}$$

which implies

$$\pi_t = p_t - p_{t-1} = m_t - m_{t-1}. \tag{4}$$

This is equivalent to assuming that the central bank controls the inflation rate perfectly (it simplifies the algebra, but is not a crucial assumption). We can then think of the policymaker as choosing the rate of inflation rather than the rate of money growth.

Equilibrium vector: $[y_t, \pi_t, \pi_t^e]$.

Rational expectations implies $\pi_t^e = E_t\pi_t$. One equation missing $\rightarrow$ The policymaker chooses monetary policy (i.e. chooses π_t) to maximize its objective function.

Two possibilities for the policymaker objective (utility) function:

1. Linear in output and quadratic in inflation (LQ)

$$U_t = \lambda(y_t - y_n) - \frac{1}{2}\pi_t^2. \tag{5}$$

   The policymaker dislikes both positive and negative deviations of inflation from zero and would like output to be as large as possible. Policymaker does not care about output variance (linear in output).

2. Quadratic in output and inflation (QQ)

$$V_t = -\frac{1}{2}\lambda(y_t - y_n - k)^2 - \frac{1}{2}\pi_t^2. \tag{6}$$

The policymaker still dislikes fluctuations in inflation around zero, but now it also dislikes fluctuations in output around $y_n + k > y_n$. This objective function generates a tradeoff between output and inflation stabilization in response to supply shocks.

In both cases, the source of the moral hazard is that while private agents' optimality requires output to be at its natural level y_n, the government targets a higher level of output possibly because y_n is inefficiently high (e.g. due to tax distortion, non-competitive product or labour markets, etc.).

(Important) Structure of the game:

Assumption: Policymaker can stabilize output in the current period because it has an informational advantage over private agents. It sets π_t after the shocks e_t is realized, while private agents cannot change their decision within the current period.

1. Agents form their expectations rationally at the beginning of time t (e.g. prices or nominal wages are set for one period in advance) before the realization of e_t. They know both the correct model of the economy (2) and the policymaker objective function (5) or (6).

2. After people have formed their expectations, e_t is realized and the policymaker chooses π_t to maximize its welfare function (5) or (6) subject to the constraint (2) *and* taking agents expectations as given. For given expectations, the choice of π_t determines the inflation rate and output level that prevail at time t.

There is a game going on between the policymaker and private sector. Relevant equilibrium concept is non-cooperative Nash (best response taking opponent's actions as given).

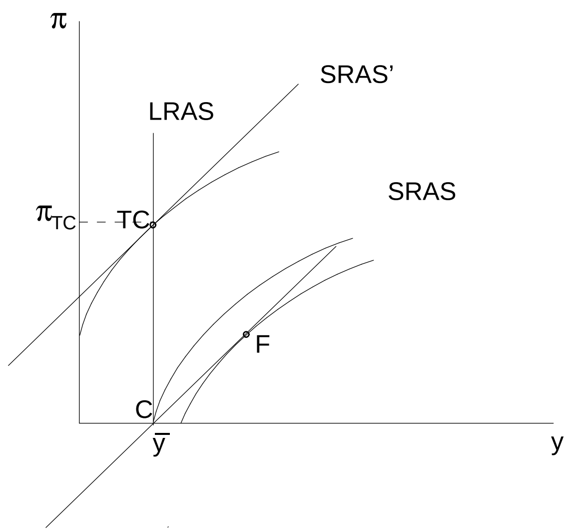
Equilibrium is a vector $[y_t, \pi_t, \pi_t^e]$ such that the AS (2) is satisfied, the government maximizes (5) or (6) subject to (2) taking expectations as given and expectations are correct ex post; i.e.

$$\pi_t^e = E_{t-1}(\pi_t) = \pi_t. \tag{7}$$

To understand the time-inconsistency problem suppose that the government announces, before people form their expectations, that it will create zero inflation at time t. Suppose there are no shocks. e_t is identically zero.

We want to show that there does not exist an equilibrium in which rational agents believe the government announcement. Suppose the contrary (i.e. that $\pi_t^e = 0$).

Notice that π_t^e determines the position of the AS. If $\pi_t^e = 0$, the SRAS passes through point C in the figure[1].



Let us proceed backward. At time t the policymaker chooses π_t optimally taking π_t^e as given. After people have formed their expectations, the government has an incentive to create unexpected inflation. At point C (i.e. if $\pi_t = \pi_t^e = 0$) the policymaker's marginal rate of substitution between inflation and output is higher than the slope of the AS: the policymaker can reach a lower indifference curve by

[1]Curves in the figure are the policymaker's indifference curves. Lower curves correspond to a higher level of welfare

creating positive unexpected inflation (point F). Since agents know the government objective function, at the beginning of time t they understand that if they believe the policymaker's pledge, the government will create positive inflation *ex post.* So, they do not believe the announcement in the first place and $\pi_t^e = 0$ is not an equilibrium unless the government has a credible way to commit to its pledge ("tie its hands").

Why is this not an equilibrium according to our definition? Rational expectations require agents to use the model to form expectations. In the absence of shocks their expectations are correct *ex post*.

Because of the policymaker's incentive to renege, expectations are realized only at a level of expected inflation such that the policymaker's MRS between inflation and output equals the slope of the SRAS (point TC). Only, if inflation is high enough the government's marginal cost and benefit from creating unexpected inflation are equal, the policymaker has no incentive to create unexpected inflation and expectations turn out to be correct.

While equilibrium output is the same at C and TC, inflation is higher at TC. There is an "inflation bias" in the time-consistent equilibrium **under discretion**.

The policymaker is strictly worse off at TC relative to C ("prisoner dilemma").

Let us now solve formally for the equilibrium. We drop the assumption that e_t is identically zero. This was necessary only for the graphical analysis.

2.2 LQ objective function (Barro-Gordon)

2.2.1 Equilibrium under discretion

The rate of inflation at time t is determined by the government optimization problem. The government chooses π_t taking private agent's expectations π_t^e and the shock realization e_t as given.

Replacing for y_t in equation (5) using (2) we obtain

$$\max_{\pi_t} U_t = \lambda(\alpha(\pi_t - \pi_t^e) + e_t) - \frac{1}{2}\pi_t^2. \tag{8}$$

The corresponding FOC is

$$\lambda\alpha - \pi_t = 0. \tag{9}$$

The policymaker has an incentive to increase inflation up to the point where the marginal benefit $\lambda\alpha$ (from higher output) exceeds the marginal cost π_t stemming from higher inflation.

The above equation gives the time-consistent rate of inflation. It is the rate of inflation which the government has no incentive to deviate from, *ex post.* It is an increasing function of α (the flatter the SRAS, the bigger the output gain from creating unexpected inflation) and increasing in λ (the bigger λ the more the government is willing to trade off higher inflation for higher output).

The third equation comes from the rational expectation assumption.

$$\pi_t^e = E_t \pi_t = \lambda \alpha. \tag{10}$$

Agents know the government welfare function and will expect the rate of inflation which *ex post* maximizes the policymaker's welfare.

Notice that as $\pi_t^e = \pi_t$, the equilibrium level of output is the full employment level y_n.

It remains to determine the ex ante (before knowing the shock realization) expected value of welfare for the policymaker.

Replacing the equilibrium values of output and inflation in equation (5) and taking expectations, the expected welfare of the policy maker in the equilibrium under discretion is

$$E[U^d] = E\left[\lambda e_t - \frac{1}{2}(\lambda\alpha)^2\right] = -\frac{1}{2}(\lambda\alpha)^2. \tag{11}$$

2.2.2 Equilibrium under (rigid) commitment

If, instead, the government were able to *credibly* commit to zero inflation, it would be $\pi_t = 0$. Given that the commitment is credible this would also be the rational expectation of inflation; i.e. $\pi_t^e = 0$. We call this *rigid* commitment because the commitment is independent from (i.e. does not allow the government to respond to) the shock e_t

The ex ante expected value of the policymaker's welfare would be

$$E[U^c] = E\left[e_t - \frac{1}{2}0\right] = 0 > E[U^d]. \tag{12}$$

2.3 QQ objective (Rogoff)

With an LQ objective the policymaker's welfare is linear in output. So the policymaker cares only about the expected value of output but not its variance. So there is no role for the government engaging in stabilization policy. It should just find a way to tie its hands and get rid of the time-inconsistency problem. Any rule, however rigid, that achieves this is optimal.

The QQ objective function instead captures the insight that there is a trade-off between rigid rules which achieves zero inflation at any cost and output stabilization.

2.3.1 Equilibrium under discretion

As before the policymaker chooses π_t taking private agent's expectations π_t^e and the shock realization e_t as given.

The only difference is that now we replacing for y_t using (2) in equation (6) rather than (5). This gives

$$\max_{\pi_t} W_t = -\frac{1}{2}\lambda(\alpha(\pi_t - \pi_t^e) + e_t - k)^2 - \frac{1}{2}\pi_t^2. \tag{13}$$

The corresponding FOC is

$$-\alpha\lambda(\alpha(\pi_t - \pi_t^e) + e_t - k) - \pi_t = 0. \tag{14}$$

Now the marginal benefit of inflation is not constant, but is decreasing in the deviation of output above its target level $y_n + k$. In particular, the supply shock now matters and implies a trade-off between inflation and output volatility. Also the policymaker's optimal choice of π_t now depends on private agents' expectations.

This can be seen by solving for π_t to obtain

$$\pi_t = \frac{\alpha^2\lambda\pi_t^e + \alpha\lambda(k - e_t)}{1 + \alpha^2\lambda}. \tag{15}$$

We can impose rational expectations by taking expectations of (14) to obtain

$$E_t\left[-\alpha\lambda(\alpha(\pi_t - \pi_t^e) + e_t - k) - \pi_t\right] = \alpha\lambda k - E_t\pi_t = 0. \tag{16}$$

Now the size of inflation bias is also increasing in k the output distortion that the government is trying to correct.

We can use equations (15) and (16) to solve for

$$\pi_t - \pi_t^e = \frac{-\pi_t^e + \alpha\lambda(k - e_t)}{1 + \alpha^2\lambda} = -\frac{\alpha\lambda e_t}{1 + \alpha^2\lambda}. \tag{17}$$

We can now solve for the policymaker's ex ante welfare by replacing on the right hand side of (13) and taking expectations to obtain

$$E[U^d] = E\left[-\frac{1}{2}\lambda\left(-\alpha\frac{\alpha\lambda e_t}{1 + \alpha^2\lambda} + e_t - k\right)^2 - \frac{1}{2}\left(\alpha\lambda k - \frac{\alpha\lambda e_t}{1 + \alpha^2\lambda}\right)^2\right] \tag{18}$$

$$= -\frac{1}{2}E\left[\lambda\left(\frac{e_t}{1 + \alpha^2\lambda} - k\right)^2 + \left(\alpha\lambda k - \frac{\alpha\lambda e_t}{1 + \alpha^2\lambda}\right)^2\right] \tag{19}$$

$$= -\frac{1}{2}\left[\lambda(1 + \alpha^2\lambda)k^2 + \frac{\lambda}{1 + \alpha^2\lambda}\sigma_e^2\right]. \tag{20}$$

2.3.2 Equilibrium under (rigid) commitment

Suppose again the policymaker could credibly commit to set $\pi_t = 0$ which, because of credibility, implies $\pi_t^e = 0$. Its expected welfare would be

$$E[U^c] = E\left[-\frac{1}{2}\lambda(e_t - k)\right] = -\frac{1}{2}\lambda(\sigma_e^2 + k^2). \tag{21}$$

Now it is not necessarily the case that (rigid) commitment yields a higher level of welfare for the policymaker.

In fact if k is low relative to σ_e^2 it is straightforward to see that $E[U^d] > E[U^c]$. The inflation bias problem is small, but the rigid commitment gives up output stabilization altogether.

3 Solutions to the inflation bias

The source of the problem is the inflation bias due to time-inconsistency.

It is the government incentive to trade-off higher inflation against higher output at when expected inflation is zero which results in the economy ending up with positive long run inflation. Unless the government is able to tie its hands the economy ends up at the inefficient equilibrium.

How can a better equilibrium be achieved?

1. Constitutional (irreversible and credible) rule which prescribes zero inflation. In our simple model this eliminates the inflation bias. Yet, we have seen such a rule is too rigid and might be suboptimal as it would rule out short run stabilization altogether (this is a general problem of any fixed rule).

2. Independent central bank. The government delegates monetary policy to an independent central bank. One possible difference between the government and the central bank is that the central bank dislikes inflation more relative to output fluctuations (has a lower λ). Assume the central banker's attaches a weight $\lambda/(1+\delta) < \lambda$ (where λ is the government weight) to output fluctuations.

   Replacing for λ in equations (16) and (17) we obtain

$$\alpha \frac{\lambda}{1+\delta} k - E_t \pi_t = 0 \tag{22}$$

and

$$\pi_t - \pi_t^e = \frac{-\pi_t^e + \alpha\lambda(k - e_t)/(1+\delta)}{1 + \alpha^2\lambda/(1+\delta)} = -\frac{\alpha\lambda e_t}{1+\delta+\alpha^2\lambda}. \tag{23}$$

We can now solve for the policymaker's ex ante welfare by replacing on the right hand side of (13) and taking expectations to obtain

$$E[U^d] = E\left[-\frac{1}{2}\lambda\left(-\alpha\frac{\alpha\lambda e_t}{1+\delta+\alpha^2\lambda} + e_t - k\right)^2 - \frac{1}{2}\left(\frac{\alpha\lambda k}{1+\delta} - \frac{\alpha\lambda e_t}{1+\delta+\alpha^2\lambda}\right)^2\right] \tag{24}$$

$$= -\frac{1}{2}E\left[\lambda\left(\frac{(1+\delta)e_t}{1+\delta+\alpha^2\lambda} - k\right)^2 + \left(\frac{\alpha\lambda k}{1+\delta} - \frac{\alpha\lambda e_t}{1+\delta+\alpha^2\lambda}\right)^2\right] \tag{25}$$

$$= -\frac{1}{2}\left[\lambda\left(\frac{(1+\delta)^2\sigma_e^2}{(1+\delta+\alpha^2\lambda)^2} + k^2\right)^2 + \left(\frac{(\alpha\lambda k)^2}{(1+\delta)^2} + \frac{(\alpha\lambda)\sigma_e^2}{(1+\delta+\alpha^2\lambda)^2}\right)\right]. \tag{26}$$

Maximizing $E[U^d]$ with respect to δ it can be shown that the optimal value of δ from the policymaker point of view is strictly positive, but less than ∞.

It is not optimal to have a central banker who cares only about inflation, since she would not care at all about output stabilization. Such a banker would set $\pi_t = 0$ and welfare would be the same as in the (rigid commitment equilibrium).

Optimal delegation to an independent central banker allows for flexibility, but the time-consistent rate of inflation is still positive, though lower.

3. Optimal contract for central bankers. The efficient equilibrium could be achieved if the government wrote an optimal contract that gave the central banker the right incentives (e.g. his/her salary is a decreasing function of the rate of inflation).

   Assume the central banker has the same preferences as the government but now her pay is given by $\tau = m + c\pi_t$.

   The central banker now maximizes $U_t + \tau$.

   In the first order condition the marginal benefit of inflation is now augmented by c.

Hence, its FOC is

$$-\alpha\lambda(\alpha(\pi_t - \pi_t^e) + e_t - k) - \pi_t + c = 0. \tag{27}$$

Imposing rational expectations we obtain

$$E_t\left[-\alpha\lambda(\alpha(\pi_t - \pi_t^e) + e_t - k) - \pi_t\right] + c = \alpha\lambda k + c - E_t\pi_t = 0. \tag{28}$$

Setting $c = -\alpha\lambda k$ sets the inflation bias to zero. While leaving everything else unchanged. It is straightforward to show, that ex ante welfare unambiguously increases.

McCallum's (1995) put forth an important criticism of these last two strands of the literature: why should the government choose a central bank with different tastes from the government itself? In other words, how can the government credibly commit not to replace the banker or not to change its contract ex post. The two latter things though may be more difficult to change ex post.

4. Reputation.