Macroeconomics A

Problem set 1

This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.

1. Consider the Solow growth model with Cobb-Douglas production function $F(K, AL) = K^{\alpha}(AL)^{1-\alpha}$. Denote by $\tilde{z} = Z/AL$ a variable Z expressed in efficiency units of labour.

   - Find expressions for the steady state equilibrium values of $\tilde{k}$, $\tilde{y}$ and $\tilde{c}$ (note that total consumption is $Y - S = (1 - s)Y$) as a function of α, s, δ, n.
   - Let z^* denote the steady state value of a variable z. Assume $\alpha = .3$, $\delta = .1$ and $n = g = .02$. Use the expression for $\tilde{y}^*$ you have derived above to obtain the values of $\tilde{y}^*$ when $s = .1$ and $s = .2$.
   - Do the same as above but with $\alpha = .8$. What can you say about the ability of the Solow model to explain cross-country income differences in terms of differences in saving rates in the two cases?
   - Derive the growth accounting equation for the Cobb-Douglas production function (Hint: you need to *derive* the capital share rather than using a generic $\alpha(t)$). Using your knowledge of the steady state rates of growth for the relevant variables verify that the growth accounting equation holds true in steady state. What proportion of per worker income growth in steady state does the equation attribute to accumulation in income per worker? What proportion to technological progress? In fact, what would be the steady state rate of growth of income per worker if the rate of technological progress were zero?

2. Consider again an economy described by the Solow growth model. Assume it is initially in steady state equilibrium. Assume the rate of technological progress is zero. Derive the effect of a one-off permanent fall in the rate of population growth on:

   - the steady state value of capital per worker, output per worker and consumption per worker;

- the time path of the (logarithm of the) above three variables as the economy moves from the old to the new steady state equilibrium (i.e. draw the logarithm of the appropriate variable against time);
- the time path of (the logarithm of) total output (not output per worker) as the economy moves from the old to the new steady state equilibrium.