

Macroeconomics A

Problem set 2

This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.

1. Consider an economy with production function

$$Y = K^\alpha T^\beta (AL)^{1-\alpha-\beta}, \quad (1)$$

where K, A, L are, as usual, capital, technological progress and labour and T is the stock of land. $\alpha, \beta > 0$. A grows at rate g and L grows at rate n . The stock of land is fixed though. Aggregate saving equals a fraction s of aggregate output.

- Derive the steady state growth rates of capital and output.
 - Derive the steady state growth rate of output per worker. Which restrictions on parameters have to be satisfied for steady state output per worker to grow at a positive rate. What is the economic intuition? (Hint: compare the degree of returns to scale to K, A, L jointly in the case in which $\beta = 0$ and $\beta > 0$ respectively.)
2. (Based on Mankiw, Romer and Weil, Quarterly Journal of Economics 1992). Assume that the production function is

$$Y = K^\alpha H^\beta (AL)^{1-\alpha-\beta},$$

where Y is output, K is physical capital, H is human capital (the skill of the labour force), L is labour and A labour-augmenting technological progress. The parameters α and β are positive and $\alpha + \beta < 1$. L and A grow at, exogenously given, constant rates n and g respectively. Output can be consumed or invested to produce either physical or human capital. Assume that gross investment in physical capital is a fraction s_K of output and that gross investment in human capital is a fraction s_H of output. Both types of capital depreciate at rate δ .

- (a) Define the economic equilibrium of the model.
- (b) You will find it useful to express all variables in efficiency units of labour; i.e. $\tilde{y} = Y/AL$, $\tilde{h} = H/AL$ and $\tilde{k} = K/AL$. Obtain an expression for $\dot{\tilde{h}}/\tilde{h}$ and $\dot{\tilde{k}}/\tilde{k}$. What are their steady state values?

- (c) Derive the steady state rate of growth of output per efficiency unit of labour \tilde{y} and output per worker y . Compare your findings to the predictions of the Solow growth model. What is the economic intuition?
- (d) Find the steady state value of the ratio h/k . Discuss the economic intuition for your findings.