Problem set 7

This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.

1. Consider the following economy with a constant population of workers normalized to $L_t = 1$ and no technological progress.
   At time t the representative household maximizes the expectation of lifetime utility, conditional on information available at time t,
$$U_t = \sum_{i=0}^{\infty} E\left(\frac{u(C_{t+i})}{(1+\rho)^i} \middle| I_t \right), \tag{1}$$
   with $u(C_{t+i}) = C_{t+i} - \theta C_{t+i}^2$, $\theta > 0$. Assume that parameters are such that $u'(C_{t+i})$ is always positive.
   The production function satisfies $Y_t = AK_t + e_t$ and capital does not depreciate. In this economy no labour is used in production and the only income for the consumer is interest income $r_{t+i}B_{t+i}$. The consumer dynamic budget constraint is the usual one but with $W_{t+i} = 0$.
   (a) Set up the consumer problem by replacing for C_{t+i} using the dynamic constraint. Derive the Euler equation.
   (b) Impose equilibrium, noticing that the marginal product of capital is A. Assume $\rho = A$ and write down the Euler equation and the dynamic budget constraint evaluated at equilibrium.
   (c) Use the Euler equation to comment on the role of consumption tilting in shaping the response of the economy to the productivity shocks e_t.
   (d) If there were no uncertainty how would you expect C_t and K_t to respond respectively to a temporary (one-off) and permanent shock e_t? What is the economy intuition behind your answer? (Hint: read the discussion in lecture notes on the response to permanent and temporary shocks in the absence of uncertainty.)
   (e) Guess that consumption takes the form $C_t = \alpha + \beta K_t + \gamma e_t$. Given this guess derive K_{t+1} as a function of K_t and e_t.
   (f) Assume e_t is white noise. This implies its expectation $E[e_{t+i}|I_t] = 0$ for all $i > 0$ (i.e. at all future times). Use K_{t+1} obtained in (e) to derive the values of α, β and γ which ensure that the Euler equation is satisfied for all values of K_t and e_t (Note: some of the parameters may be zero). Derive the effect of a one-off positive shock e_t on the paths of Y_t, K_t and C_t.
   (g) Assume e_t is a random walk. This implies $e_{t+1} = e_t + v_{t+1}$ with v_{t+1} white-noise and $[e_{t+i}|I_t] = e_t$ for all $i > 0$. Use K_{t+1} obtained in (e) to derive the values of α, β and γ which ensure that the Euler equation is satisfied for all values of K_t and e_t (Note: some of the parameters may be zero). Derive the effect of a one-off positive shock e_t on the paths of Y_t, K_t and C_t. Compare this result to that in (f) above.