

## Macroeconomics A

### Problem set 9

*This problem set will be collected at the beginning of the next class and marked. If you cannot make it to class you are advised to hand your solutions in before the deadline. No solution will be accepted after the deadline.*

1. Consider the following problem. The aggregate supply equation is given by

$$y_t = (a + e_t)(\pi_t - \pi_t^e), \quad (1)$$

where  $y$  is output,  $\pi$  is actual inflation and  $\pi^e$  the private sector's expected inflation;  $a$  is a positive constant and  $e$  an independent and non-autocorrelated random shock with zero mean and variance  $\sigma^2$ . The monetary authority sets the rate of inflation after observing the shock  $e$ . Private sector expectations are rational and formed before the authorities determine actual inflation. Private agents never observe  $e$ . The policymaker welfare function is given by

$$W = \lambda y - \frac{\pi^2}{2}, \quad (2)$$

with  $\lambda > 0$ .

- (a) What is the expected inflation rate under discretion? What is the expected value of  $W$ ?
- (b) Suppose that the authorities could commit, before the realization of  $e$ , to follow the linear rule  $\pi = c + de$ . What are the optimal values of  $c$  and  $d$ ? What is the corresponding expected level of  $W$  under this optimal rule? Why might this rule be difficult to implement in practice?
- (c) Suppose that instead the authorities decide to appoint an independent central banker with preferences

$$V = \mu y - \frac{\pi^2}{2}, \quad (3)$$

where  $\mu > 0$ . What will be the inflation rate and the associated expected level of government welfare  $W$ ? What value of  $\mu$  maximizes expected government welfare  $W$ ? Why is it different from zero? How does expected  $W$  under delegation compares with its value in part (b)?