Macroeconomics A

Solution to problem set 6

By assumption, it is $\delta = n = 0$.

In intensive units the consumer problem is

$$U_0 = \max_{c_t} \int_0^\infty \frac{c_t^{1-\theta}}{1-\theta} e^{-[\rho-(1-\theta)g]t} dt \tag{1}$$

$$\text{s.t. } \dot{b}_t = (r_t - g)b_t + w_t + S_t - c_t \tag{2}$$

$$\lim_{t\to\infty} b_t e^{-R_t+gt} \geq 0 \tag{3}$$

$$b_0 \text{ given,} \tag{4}$$

where S_t is the government transfer.

Equation (2) and (3) can be used to derive the intertemporal budget constraint and write the Lagrangean associated with the consumer problem as

$$\mathcal{L} = \int_0^\infty \frac{c_t^{1-\theta}}{1-\theta} e^{-[\rho-(1-\theta)g]t} dt + \lambda \left[b_0 + \int_0^\infty (w_t + S_t - c_t) e^{-R_t+gt} dt \right]. \tag{5}$$

The sequence of FOCs, one for each t, is

$$c^{-\theta} e^{-[\rho-(1-\theta)g]t} = \lambda e^{-R_t+gt}, \tag{6}$$

which simplifies to

$$c^{-\theta} = \lambda e^{-[R_t-(\rho+\theta g)t]}. \tag{7}$$

Taking logs and time derivatives we obtain the Euler equation

$$\frac{\dot{c}}{c} = \frac{r_t - \rho - \theta g}{\theta}. \tag{8}$$

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(a) Given CRS firms maximize unit profits

$$\Pi_t = f(k_t) - R_t k_t - w_t, \tag{9}$$

where R_t is the price at which firms rent capital. The FOC for capital is $f'(k_t) = R_t$ and the zero profit condition is $w_t = f(k_t) - R_t k_t = f(k_t) - f'(k_t)k_t$.

Factor market equilibrium implies $b_t = k_t$ and by definition the net return on capital accruing to the consumer equals the cost to the firm net of taxes; i.e. $r_t = R_t(1-\tau)$. Replacing in (8) the latter implies

$$\frac{\dot{c}}{c} = \frac{(1-\tau)f'(k_t) - \rho - \theta g}{\theta}. \tag{10}$$

. While replacing for S_t, r_t and w_t in the dynamic constraint we obtain

$$\dot{k}_t = ((1-\tau)f'(k_t) - g)k_t + \underbrace{f(k_t - f'(k_t)k_t}_{w_t} + \underbrace{\tau f'(k_t)k_t}_{S_t} - c_t = \tag{11}$$

$$= f(k_t) - gk_t - c_t. \tag{12}$$

So only the first equation is affected by the policy. The $\dot{c} = 0$ locus shifts left. The transfer leaves income unchanged, but the tax distorts the intertemporal marginal rate of substitution.

(b) The economy jumps on the saddle path going through the new steady state. Consumption jumps up and then both consumption and capital converge to their new steady state values.

(c) Both c and k are lower in the new steady state

(d) (i) Using the steady state accumulation equation the saving rate is

$$\frac{f(k^*) - c^*}{f(k^*)} = \frac{gk^*}{f(k^*)}. \tag{13}$$

Since k^* has fallen, the average product of capital $f(k^*)/k^*$ is higher and the saving rate is lower in the new steady state.

(ii) The Euler equation (8) implies that in steady state the net of tax interest rate r_t is equalized across countries and equals $r_t = \rho + \theta g$ independently from the equilibrium value of k^*. So there is no incentive for capital to flow as long as the return to capital is taxed in the country in which it is produced (taxation at source rather than in the country of destination).

(e) No, because the decentralized equilibrium is Pareto optimal as it is a competitive equilibrium of an economy which satisfies all the conditions of the first welfare theorem.

(f) The Euler equation (10) is unchanged but now $S_t = 0$ in the intertemporal budget constraint. Therefore the accumulation constraint is now

$$\dot{k}_t = ((1-\tau)f'(k_t) - g)k_t + \underbrace{f(k_t) - f'(k_t)k}_{w_t} - c_t = f(k_t) - gk_t - c_t - \tau f'(k_t)k_t, \tag{14}$$

which is lower than (11).