Macroeconomics A

Solution to problem set 7

I have made a mistake in setting up this problem. Given the chosen production function, assuming that the only income is income from renting capital to firms leaves a residual e_t which is unassigned. This is bad! Assume that e_t is distributed as dividend income to consumers. Let us denote by D_t dividend income.

1. The dynamic budget constraint is now

$$B_{t+1} = (1 + r_t)B_t + D_t - C_t \tag{1}$$

as there is no labour income.

(a) Replacing for $C_t + i$ in the consumer problem using the dynamic constraint gives

$$\max_{\{B_{t+i+1}\}_{i=0}^{\infty}} \sum_{i=0}^{\infty} E\left(\frac{u(B_{t+i+1} + D_t - (1 + r_t)B_t)}{(1+\rho)^i} \Big| I_t \right). \tag{2}$$

The first order condition with respect to B_{t+1} is

$$u'(C_t) = E\left(\frac{1 + r_{t+1}}{1+\rho} u'(C_{t+1}) \Big| I_t \right). \tag{3}$$

Given the utility function it is $u'(C) = 1 - 2\theta C$ and the Euler equation (3) can be rewritten as

$$1 - 2\theta C_t = E\left(\frac{1 + r_{t+1}}{1+\rho} (1 - 2\theta C_{t+1}) \Big| I_t \right). \tag{4}$$

(b) Equilibrium requires $r_{t+1} = F'(K_{t+1}) = A$, where A is deterministic. Hence, in equilibrium the Euler equation is

$$1 - 2\theta C_t = E\left(\frac{1 + A}{1+\rho} (1 - 2\theta C_{t+1}) \Big| I_t \right) \tag{5}$$

and, given $\rho = A$,

$$1 - 2\theta C_t = E\left((1 - 2\theta C_{t+1}) \Big| I_t \right) \tag{6}$$

or

$$C_t = E\left(C_{t+1} | I_t\right). \tag{7}$$

Finally, in equilibrium, the dynamic budget constraint becomes

$$K_{t+1} = (1 + A)K_t + e_t - C_t. \tag{8}$$

Now things add up!

(c) With quadratic preferences and linear production function with additive shocks, there is no consumption tilting effect on the Euler equation as shocks do not affect the ratio between $1+r_{t+1}$ and $1+\rho$. So all the dynamics will be driven by consumption smoothing.

(d) Without uncertainty, it is $C_t = C_{t+1}$. Consumption is flat. Consumption would respond one-to-one to an unexpected permanent shock. On the other hand, it will respond less than one-to-one to the permanent shock to spread the one-off shock across the whole lifetime. The intertemporal budget constraint requires the present value of the increase in consumption to equal the size of the one-off shock e_t.

(e) Use the guess to replace for C_t in the equilibrium dynamic constraint (8). This gives

$$K_{t+1} = (1+A)K_t + e_t - (\alpha + \beta K_t + \gamma e_t). \tag{9}$$

with $\alpha,\ \beta,\ \gamma$ to be determined.

(f) The guess must also satisfy the Euler equation which requires

$$\alpha + \beta K_t + \gamma e_t = E\left(\alpha + \beta K_{t+1} + \gamma e_t | I_t\right) \tag{10}$$

or

$$\beta K_t + \gamma e_t = \beta E(K_{t+1}|I_t) + \gamma E(e_t|I_t). \tag{11}$$

Using the policy function for K_{t+1} in (9) this becomes

$$\beta K_t + \gamma e_t = \beta E\big(\alpha + (1+A-\beta)K_t + (1-\gamma)e_t)\big|I_t\big) + \gamma E(e_t|I_t). \tag{12}$$

This is now a deterministic equation which can be solved for the unknown parameters $\alpha,\ \beta,\ \gamma$. The value of $E(e_{t+1}|I_t)$ depends on how e_t is distributed.
If e_t is white noise, it is $E\left[e_{t+1}|I_t\right] = 0$ and (13) becomes

$$\beta K_t + \gamma e_t = \beta\big(\alpha + (1+A-\beta)K_t + (1-\gamma)e_t)\big). \tag{13}$$

For it to be satisfied for any K_t and e_t we must equate coefficients on the same variables on both sides of the equation. This requires $\alpha = 0$, $\beta = A$ and $\gamma = A(1-\gamma)$ or $\gamma = A/(1+A)$. Replacing in equation (9) gives

$$K_{t+1} = K_t + \frac{1}{1+A} e_t \tag{14}$$

and

$$C_t = K_t + Y_t - K_{t+1} = (1+A)K_t + \left(1 - \frac{A}{1+A}\right) e_t = (1+A)K_t + \frac{A}{1+A} e_t. \tag{15}$$

The shock is temporary, there is no consumption tilting. Consumption smoothing dictates that most of the shock is save (i.e. goes into $K_{t+1} - K_t$) and only a fraction is consumed.

(g) In this case it is $E(e_{t+1}|I_t) = e_t$. Replacing in equation (13) we obtain

$$\beta K_t + \gamma e_t = \beta\big(\alpha + (1 + A - \beta)K_t + (1 - \gamma)e_t)\big) + e_t. \tag{16}$$

For it to be satisfied for any K_t and e_t we must equate coefficients on the same variables on both sides of the equation. This requires $\alpha = 0$, $\beta = 1 + A$ and $\gamma = 1$. Replacing in equation (9) gives

$$K_{t+1} = K_t \tag{17}$$

and

$$C_t = K_t + Y_t - K_t + 1 = Y_t = AK_t + e_t. \tag{18}$$

The shock is permanent, there is no consumption tilting. Consumption smoothing dictates that all the of the shock is consumed and none of it is saved.