Macroeconomics A

Solution to problem set 8

Solving for the equilibrium vector $[y_t, p_t, Ep_t]$ is done always in the same way.
Substitute for y_t using the AS curve in AD to obtain

$$\alpha \left(p_t - E_{t-1}p_t\right) = m_t - p_t + v_t \tag{1}$$

Taking expectations of both sides gives

$$0 = E_{t-1}m_t - E_{t-1}p_t + E_{t-1}v_t \tag{2}$$

which gives $E_{t-1}p_t$ as a function of exogenous variables
Subtract (2) from the AD curve to obtain

$$p_t - E_{t-1}p_t = (m_t - E_{t-1}m_t) + (v_t - E_{t-1}v_t) - y_t. \tag{3}$$

Replacing for $p_t - E_{t-1}p_t$ in the AS and rearranging gives you the equilibrium value of output as a function of exogenous variables

$$y_t = \frac{\alpha}{1+\alpha}\left[(m_t - E_{t-1}m_t) + (v_t - E_{t-1}v_t)\right]. \tag{4}$$

Finally, replacing for y_t in (3) using (4) and rearranging gives $p_t - E_{t-1}p_t$ as a function of exogenous variables (This last step is just for your knowledge. you do not need this last bit to answer this problem set)

$$p_t - E_{t-1}p_t = \frac{1}{1+\alpha}\left[(m_t - E_{t-1}m_t) + (v_t - E_{t-1}v_t)\right]. \tag{5}$$

We can now use the general result to answer the individual questions.

1. It is $E_{t-1}v_t = v_{t-2}$ and $E_{t-1}m_t = \bar{m} + (\gamma_0 + \gamma_1)v_{t-2}$. Replacing in equation (4) gives

$$y_t = \frac{\alpha}{1+\alpha}\left\{[\gamma_0(v_{t-1} - v_{t-2}) + \gamma_1(v_{t-2} - v_{t-2})] + (v_{t-1} + \epsilon_t - v_{t-2})\right\} \tag{6}$$

$$= \frac{\alpha}{1+\alpha}[(1+\gamma_0)\epsilon_{t-1} + \epsilon_t]. \tag{7}$$

Remembering that ϵ_t is serially uncorrelated we obtain

$$Var\{y\} = \frac{\alpha^2}{(1+\alpha)^2}[(1+\gamma_0)^2\sigma_\epsilon^2 + \sigma_\epsilon^2] \tag{8}$$

which is minimized for $\gamma_0 = -1$ and any value of γ_1.

Intuition: since v_{t-2} is observed by workers, any systematic response to it is fully anticipated by workers and has no impact on output.

2. It is now $E_{t-1}v_t = 0$ and $E_{t-1}m_t = \bar{m}$. Using again equation (6) but the new values for the policy rule and the shocks v_t and u_t results in

$$y_t = \frac{\alpha}{1+\alpha}\left[(-\gamma u_t) + (v_t)\right] = \frac{\alpha}{1+\alpha}\left[-\gamma(v_t + e_t) + (v_t)\right]. \quad (9)$$

The output variance is then

$$Var\{y\} = \frac{\alpha^2}{(1+\alpha)^2}[\gamma^2(\sigma_v^2 + \sigma_e^2) + \sigma_v^2 - 2\gamma\sigma_v^2] \quad (10)$$

which is minimized for $\gamma = \sigma_v^2/(\sigma_v^2 + \sigma_e^2)$. Intuition: signal extraction problem as in Lucas island model. If $\sigma_e = 0$, observing u_t would be equivalent to observing v_t and the policy maker could offset the shock perfectly (given its informational advantage) by setting $\gamma = 0$. But if $\sigma_e^2 > 0$ the optimal response to u_t is less than one-for-one since part of the fluctuation in u_t is just noise in the signal without any effect on output.

3. The graphical discussion will be done in class. For the effect of fiscal policy we know that an anticipated temporary change in government expenditure affects the real interest rate, capital accumulation and output. So the Lucas-Sargent-Wallace policy ineffectiveness proposition does not apply in this case.

   An anticipated permanent change in government expenditure instead does not affect output or the real interest rate.