ECOM 009 Macroeconomics B

Lecture 2

Giulio Fella

Aim of consumption theory

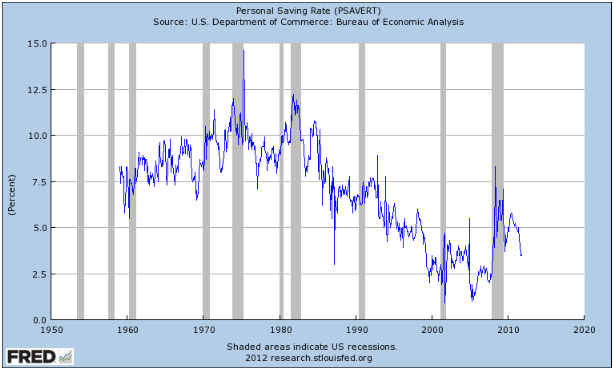
Consumption theory aims at explaining consumption/saving decisions to answer questions like:

- How does consumption relate to income and income uncertainty?
- What drives wealth accumulation and decumulation (i.e. why do people save)?
- How do changes in the interest rate or the rate of output growth affect saving?
- How do life-cycle factors (family size, hours worked, length of working life) affect consumption/saving decisions?
- How do individual consumption decisions aggregate into aggregate consumption?

Some stylized facts and some puzzles

Stylized facts (aggregate data)

Personal saving rate $= 1 - \frac{C}{Y_d}$



8 CHAPTER 2. SOME STYLIZED FACTS AND SOME PUZZLES

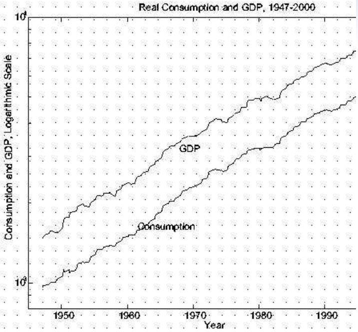


Figure 2.3: Levels of Real Consumption Expenditures and GDP

Figure : Source: Krueger 2005

1. Aggregate consumption and income grow at roughly the same rate over time. → share roughly constant.
2. Consumption is less variable than income.

3. Aggregate consumption represents between 60 and 70% of GDP.
4. Aggregate saving rates are positively correlated with the rate of output growth.

Stylized facts (individual data)

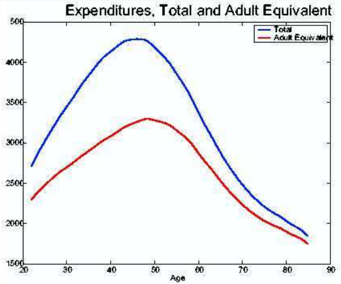


Figure : Source: Krueger 2005

1. Both consumption and income follow a hump-shaped pattern over the life cycle. Consumption tracks income over the life cycle.
2. Consumption falls at retirement.

Puzzles

Puzzles for the intertemporal theory of consumption

- Excess sensitivity puzzle: consumption responds "too much" to *predictable* changes in income.
  Related:
  - Retirement consumption puzzle: consumption falls "too much" at retirement
  - Lack of decumulation puzzle: very old households still hold a "too large" stock of financial wealth
- Excess smoothness puzzle: if income has a unit root, consumption responds "too little" to *unexpected* changes (innovations) in income

(Puzzling) early theories of consumption

1. Keynesian theory of aggregate consumption. Aggregate consumption is a stable function $C_t = \bar{C} + cY_t$, $0 < c < 1$ of current income ("a higher absolute level of income ... will lead... to a greater *proportion* of income being saved" Keynes 1936).
   Problems (see Romer 7.1):
   - Aggregate time series data: C/Y is roughly constant.
   - Cross sectional data: intercept of consumption function differs over time and across groups.

Solutions

2. Irving Fischer's intertemporal theory. Individual consumption/saving choices reflect a trade-off between present and future consumption
3. Friedman's Permanent Income theory. Saving is a way to keep a smooth consumption profile in the face of uncertain income. Individual fully adjust their consumption in response to permanent changes in income, but increase (reduce) it only partially in response to positive (negative) temporary income shocks.
4. Modigliani-Brumberg Life Cycle Theory. Saving is a way to smooth consumption in the face of a non-smooth income profile over the life cycle (labour income initially increases over the working life and falls at retirement).

Main drivers of saving

Modern consumption theory

- Intertemporal choice. It provides microfoundations for Friedman's and Modigliani-Brumberg's theories by casting them in Fisher's intertemporal optimization framework.
- Various versions: 1) finite vs infinite lifetimes; 2) perfect insurance vs only self-insurance through riskless borrowing/lending; 3) exogenous vs endogenous income process; 4) partial vs general equilibrium.

A simple two period model

Consider a consumer with 2 period lifetime which can borrow and lend at the riskless rate r. The consumer maximizes

$$\max_{c_1,c_2,a_2,a_3} u(c_1) + \beta E_1 u(c_2) \tag{17}$$

$$\text{s.t. } a_{t+1} = (1+r)\,a_t + y_t, \; t = 1, 2 - c_t \tag{18}$$

$$a_1 = 0, a_3 \geq 0, \tag{19}$$

with $\beta = 1/(1+\rho) < 1$, $u_c > 0, u_{cc} < 0$, $u_c(0) = \infty$.
E_1 is the expectation operator conditional on all information available at time 1.
We allow for income to be stochastic.

Given non-satiation the solvency constraint must hold as an equality. $a_3 = 0 \to 0 = (1+r)a_2 + y_2 - c_2$
The FOC (Euler equation) is

$$u'(c_1) = \beta (1+r) E_1 u'(c_2). \tag{20}$$

This is the *Euler equation* for the consumption problem.

- Expected marginal utility of consumption discounted at the market interest rate has to be equalized across the two periods.
- Statement on the *slope* of expected marginal utility over time not its level.
- THE testable implication of intertemporal consumer theory.

(1) Consumption tilting motive

Assume no uncertainty and $y_1 = y_2$. The Euler equation

$$u_c(c_1) = \beta(1+r)\,u_c(c_2)\,. \tag{21}$$

implies that the consumption profile is upward sloping if $\beta(1+r) > 1$; i.e. if individuals subjective discount rate is lower than the market rate of return on saving. Viceversa if $\beta(1+r) < 1$.

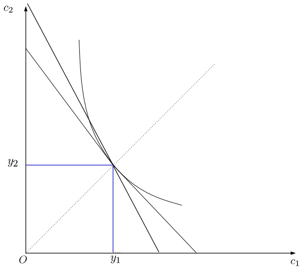


Figure : Consumption tilting

(2) Consumption smoothing motive

Assume $\beta(1+r) = 1$ (no consumption tilting motive) and income is stochastic.
The Euler equation becomes

$$u'(c_1) = E_1 u'(c_2). \tag{22}$$

Second period consumption and utility are uncertain given that y_2 is unknown as of time 1.
If $u(c_t) = -(c_t - \bar{c})^2/2$, which is increasing as long as $c_t < \bar{c}$ (we assume $\bar{c}$ is so large that it is never attained), the Euler equation is

$$c_1 = E_1 c_2. \tag{23}$$

- Individuals behave as if there were no uncertainty (certainty equivalence). This is because u'' is constant.
- No effect of income uncertainty on saving behaviour (but yes on utility!).
- Saving is positive (negative) if $y_1 > E_1 y_2$ ($y_1 < E_1 y_2$).

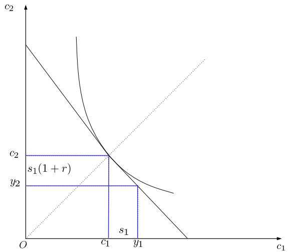


Figure : Consumption smoothing

Taking stock

Hump-shaped pattern of consumption over the lifetime cannot be explained by either consumption smoothing or tilting.

- Consumption smoothing implies flatness
- Consumption tilting implies monotonicity over time

(3) Precautionary saving

Suppose it is still $\beta(1+r)=1$, but now the utility function has $u'''>0$; i.e. u' is convex. Such a function is said to display *prudence.*

Expected marginal utility is increasing in consumption uncertainty.

Even if $y_1 = E_1 y_2$, Jensen inequality implies

$$u'(y_1) < E_1 u'(y_2). \tag{24}$$

If individuals set $c_t = y_t$, the marginal utility of consumption in period 1 is below the expected marginal utility in period 2. Hence, consumption needs to fall in period 1 (saving needs to be positive for optimality). The higher income uncertainty the larger *precautionary* saving in the first period.

Intuition: consider two possible lotteries over period 2 consumption.

- Lottery 1: $c_2 = c_1 + \begin{cases} c_1/4 \text{ with prob. } 0.5 \\ -c_1/4 \text{ with prob. } 0.5 \end{cases}.$
- Lottery 2: $c_2 = c_1 + \begin{cases} c_1/2 \text{ with prob. } 0.5 \\ -c_1/2 \text{ with prob. } 0.5 \end{cases}.$

Both imply $c_1 = E_1 c_2$. But the second one implies a lower value of c_2 in the bad state.

A consumer with quadratic preference is indifferent between the two lotteries.
A prudent consumers prefers the first one. Going from lottery 1 to 2 the marginal utility of consumption in the bad state increases by more than it decreases in the good state.
For given average consumption in period 2, expected MU is increasing in consumption uncertainty.

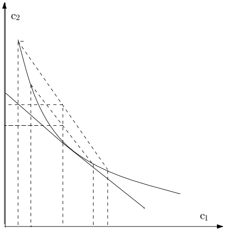


Figure : Convex versus linear MU

As income uncertainty increases c_1 falls (saving increases) as the consumer tries to insure against the worst possible consumption realization.

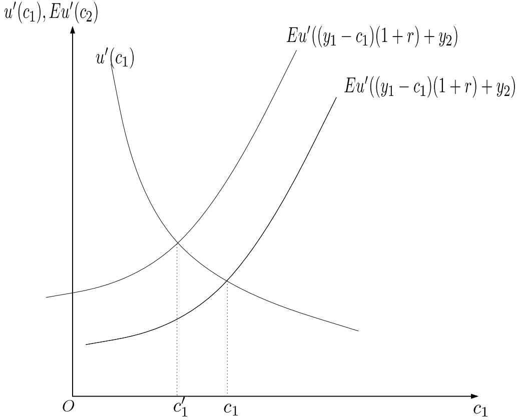


Figure : Precautionary saving

Summary

We want to study the relationship between consumption and saving on the one hand and income and income uncertainty on the other. The main two reasons behind saving we will consider are:

- Smoothing consumption in the face of an uncertain (non-smooth) income profile. The role of expectations is crucial. Changes in income induce individuals to revise their expectations.
- Precautionary saving. Saving to avoid ex post consumption variability.

Common (**crucial**) assumption: RE. Agents' expectations with the mathematical expectations $E[.|I_t]$ with I_t the consumer's information set. $\rightarrow$ for any variable z_t it is
$z_{t+1} = E_t[z_{t+1}|I_t] + u_{t+1}$ with $E_t[u_{t+1}|I_t] = 0$.

The Permanent Income model

Permanent income consumption hypothesis (PICH-LQ)

Studies consumption/saving decisions under the following assumptions:

- Infinite lifetimes (no life cycle);
- Exogenous income uncertainty (leisure/labour choice is ruled out);
- No insurance markets. Only one asset market for a riskless bond, which pays a sure and constant return r. Consumers can freely borrow and lend (self-insure) at the rate r subject to solvency.
- Partial equilibrium. Exogenous income process and risk-free rate.
- Linear quadratic (LQ) problem; i.e. Quadratic preferences. $\rightarrow$ **No precautionary saving,** but closed form solution.

Sequence problem (SP)

$$\max_{\{c_s,a_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} E_t u(c_s) \tag{25}$$

$$\text{s.t. } a_{s+1} = (1+r)\,a_s + y_s - c_s,$$

$$a_t \text{ given, } \lim_{s\to\infty} \frac{a_s}{(1+r)^s} \geq 0.$$

Recursive problem (RP)

$$W(a_s, z_s) = \max_{\{c_s,a_{s+1}\}} u(c_s) + \beta E_s W(a_{s+1}, z_{s+1}) \tag{26}$$

$$\text{s.t. } a_{s+1} = (1+r)\,a_s + y_s - c_s,$$

$$a_t \text{ given, } \lim_{s\to\infty} \frac{a_s}{(1+r)^s} \geq 0,$$

where z_s is an "appropriately" chosen state variable(s).

Whether we obtain the FOC from SP or (more easily) from RP if the maximum is internal the envelope condition implies the Euler equation

$$u'(c_s) = \beta(1+r)E_s u'(c_{s+1}). \tag{27}$$

- The condition is necessary (and sufficient given concavity) for a maximum.
- Hall (1978): main implication of optimization and RE. (Discounted) expected marginal utility is a martingale.
- **Current marginal utility is best predictor of (and sufficient statistic for) future marginal utility.**

Equivalently,

$$\beta(1+r)u'(c_{s+1}) = u'(c_s) + \epsilon_{s+1}, \quad E_s[\epsilon_{s+1}] = 0. \tag{28}$$

- Any deviations of (discounted) actual marginal utility from its expectation is orthogonal to any variable in the consumer information set at time t, I_s.
- Assuming a functional form for u, allows one to test the hypothesis by running a regression like (28) including variables which are in I_s.
- The predictions of the theory (toghether with the maintained functional form assumption on utility) are not rejected if one does not reject the hypothesis that the coefficients associated with those variable are zero.

- The Euler equation (27) is important not only theoretically. Together with RE it implies moment restrictions on the covariance between the ratio of marginal utility and variables in I_s.
- These moment restrictions can be used for (GMM) estimation of the parameters in the Euler equation (Hansen and Singleton 1982)

From the Euler eq. to the consumption function

- Euler equation characterizes only the marginal utility *profile* (somewhat related to the consumption profile). We need a consumption fn. (levels) if we want e.g. to know the effect of a change in r on consumption.
- We need to solve for the **level** of consumption (consumption function). A closed-form solution exists only in a few special cases. To simplify the algebra, we make the additional (non-crucial) assumption $\beta(1+r)=1$.

Recipe to solve for consumption function:

1. Set up RP or SP (specify solvency constraint) and derive the Euler equation.
2. Since the Euler equation is the same in both cases, two alternatives:
   2.1 SP: (i) integrate dynamic constraint; (ii) impose solvency $\rightarrow$ intertemporal budget constraint (IBC); (iii) substitute for futures consumption as function of present consumption in IBC and solve for present consumption.
   2.2 RP: (i) guess a state variable z_t for the stochastic process for income; (ii) guess a stationary policy function $c(a_t, z_t)$; (iii) use the Euler equation and the dynamic budget identity to solve for the unknown parameters of your guess.

$\beta(1+r)=1$ implies the Euler equation is:

$$u(c_s) = E_s u'(c_{s+1}), \quad \text{for all } s. \tag{29}$$

For a general utility function, the consumption function does not have closed form solution.

- **quadratic utility**: $u(c_s) = -(c_s - \bar{c})^2/2$

VERY special case: not only closed form solution, but it resembles the certainty case very closely. Linear marginal utility.

Euler equation becomes

$$c_s = E_s c_{s+1}. \tag{30}$$

- Quadratic utility implies $c_{s+1} = c_s + \epsilon_{s+1}$ with $E_s \epsilon_{s+1} = 0$. $\rightarrow$ Changes in consumption (not only in marginal utility) are innovations.
- Equation (30) holds for any s and we can write

$$E_t(c_s - E_s c_{s+1}) = 0 \rightarrow E_t c_s = E_t c_{s+1}, \;\; s \geq t \tag{31}$$

  by the law of iterated expectations.
- Therefore for any s it is

$$c_t = E_t c_s, \;\; s \geq t. \tag{32}$$

General case (SP)

Solving for an arbitrary income process can only done for the sequence problem.
The intertemporal budget constraint must still hold at any t with probability one if the no-Ponzi-game constraint is satisfied (i.e. if we impose solvency). If it holds with probability one it also holds in expected value and we can write

$$\sum_{s=t}^{\infty} \frac{E_t c_s}{(1+r)^{s-t}} = (1+r)a_t + \sum_{s=t}^{\infty} \frac{E_t y_s}{(1+r)^{s-t}}. \tag{33}$$

Noticing that $E_t c_s = c_t$ hence is non-random we can write

$$c_t = r\left(a_t + H_t\right) = Y_t^p, \tag{34}$$

where now $H_t = (1+r)^{-1} \sum_{s=t}^{\infty} E_t y_s/(1+r)^{s-t}$.

Implications:

- Marginal propensity to consume out of financial wealth is small.

  From (34) the marginal propensity to consume out of unexpected changes (windfalls) in wealth is $r \sim 0.03$.

- Marginal propensity to consume out of labour income depends on the stochastic process for income. It depends on how current income relates to expected future income.

Determinants of the consumption innovation

- The Euler equation implies that changes in consumption are a pure innovation (unforecastable on the basis of I_t).
- Adding the intertemporal budget constraint, implies an additional prediction. Equation (34) makes clear that they are not *any* innovation, they coincide with innovations in *permanent* income. It is

$$c_{t+1} - c_t = Y^p_{t+1} - E_t Y^p_{t+1} = r(a_{t+1} + H_{t+1}) - rE_t(a_{t+1} + H_t) =$$

$$r(H_{t+1} - E_t H_{t+1}) = \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{E_{t+1} y_{s+1} - E_t y_{s+1}}{(1+r)^{s-t}}, \qquad (35)$$

where the first equality follows from (30) and (34) and the third one from the fact that a_{t+1} is known at time t.

$$c_{t+1} - c_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \frac{E_{t+1} y_{s+1} - E_t y_{s+1}}{(1+r)^{s-t}}$$

- Consumption responds only to *news or surprises,* i.e. shocks that induce the agent to revise her expectation about permanent income (or lifetime labour income).
- **Optimization +RE implies that the stochastic process for labour income imposes restrictions on the stochastic process for consumption.**

What about saving?

Use again (34) to obtain

$$c_{t+1} - c_t = r(a_{t+1} + H_{t+1}) - r(a_t + H_t) = r(H_{t+1} - E_t H_{t+1}) \quad (36)$$

which yields

$$\begin{aligned} s_t = & a_{t+1} - a_t = H_t - E_t H_{t+1} = \\ & \frac{1}{1+r} \sum_{s=t}^{\infty} - \frac{E_t(y_{s+1} - y_s)}{(1+r)^{s-t}} = \frac{1}{1+r} \sum_{s=t}^{\infty} - \frac{E_t \Delta y_{s+1}}{(1+r)^{s-t}}. \end{aligned}$$

What about saving?

Use again (34) to obtain

$$c_{t+1} - c_t = r(a_{t+1} + H_{t+1}) - r(a_t + H_t) = r(H_{t+1} - E_t H_{t+1}) \quad (36)$$

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- Saving-for-a-rainy-day equation. Saving is positive if current lifetime labour income exceeds tomorrows expected value.
- Cfr 2 period model w/ quadratic utility: saving was positive if $Y_1 > EY_2$.

Specific income process (RP)

Permanent and temporary income shocks.

We now derive the response of consumption and saving to income shocks under different assumptions about the stochastic process for income. SP is much easier to solve.

Suppose income can be decomposed into a permanent and temporary component:

$$y_t = y_t^p + u_t$$

and

$$y_t^p = y_{t-1}^p + \psi_t$$

with ψ_t and u_t white noise.

u_t is a purely temporary component. y_t^p is a random walk.

Guess $z_t = (y_t^p, u_t, \psi_t)$ (why?) and the consumption function

$$c(a_t, y_t^p, u_t, \psi_t) = \alpha_0 + \alpha_1 a_t + \alpha_2 y_t^p + \alpha_3 u_t + \alpha_4 \psi_t. \tag{37}$$

Replacing in the Euler equation $c_t = E_t(c_{t+1})$ gives

$$\alpha_0 + \alpha_1 a_t + \alpha_2 y_t^p + \alpha_3 u_t + \alpha_4 \psi_t = \alpha_0 + \alpha_1 a_{t+1} + \alpha_2 y_t^p \tag{38}$$

or

$$a_{t+1} - a_t = \frac{\alpha_3}{\alpha_1} u_t + \frac{\alpha_4}{\alpha_1} \psi_t. \tag{39}$$

Replacing for c_t and y_t in the dynamic budget identity yields

$$a_{t+1} - a_t = r a_t + y_t^p + u_t - (\alpha_0 + \alpha_1 a_t + \alpha_2 y_t^p + \alpha_3 u_t + \alpha_4 \psi_t). \tag{40}$$

Equating the RHS of (39) and (40) and solving for the unknown coefficients α_i yields

$$c_t = ra_t + y_t^p + \frac{r}{1+r} u_t. \tag{41}$$

$$c_{t+1} - c_t = c_{t+1} - E_t c_{t+1} = \psi_{t+1} + \frac{r}{1+r} u_{t+1}. \tag{42}$$

The marginal propensity to consume out of a transitory shock is roughly r. The marginal propensity to consume out of permanent shocks is 1.

What happens to saving?

$$s_t = a_{t+1} - a_t = ra_t + y_t^p + u_t - c_t = \frac{1}{1+r} u_t. \qquad (43)$$

Most of the transitory shock is saved to spread it over all future periods. None of the permanent one is.

Specific income process (RP)

Persistent income shocks.

What if consumers cannot distinguish between permanent shocks (ψ_t) and transitory ones (ψ_t)? Suppose income shocks are persistent and follow an AR(1) process.

$$y_t = \mu(1-\lambda) + \lambda y_{t-1} + \epsilon_t \tag{44}$$

with $E(\epsilon_t) = 0$

Guess $z_t = (y_t, \epsilon_t)$ and $c_t = \alpha_0 + \alpha_1 a_t + \alpha_2 y_t + \alpha_3 \epsilon_t$. Replacing in the Euler equation yields

$$a_{t+1} - a_t = \frac{\alpha_2}{\alpha_1}(1-\lambda)[y_t - \mu] + \frac{\alpha_3}{\alpha_1}\epsilon_t. \tag{45}$$

Replacing for c_t in the dynamic constraint

$$a_{t+1} - a_t = r a_t + y_t - (\alpha_0 + \alpha_1 a_t + \alpha_2 y_t + \alpha_3 \epsilon_t) \tag{46}$$

Equating (45) and (46) yields

$$c_t = ra_t + \frac{1}{1-\lambda+r}[\mu(1-\lambda) + ry_t] \tag{47}$$

Which implies

$$c_{t+1} - c_t = c_{t+1} - E_t c_{t+1} = \frac{r}{1-\lambda+r}(y_{t+1} - E_t y_{t+1}) = \frac{r}{1-\lambda+r}\epsilon_{t+1}.$$

The purely transitory and purely permanent shock cases are the limit cases when $\lambda = 0$ and $\lambda = 1$.