ECOM 009 Macroeconomics 9

Lecture 9

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Plan for this lecture

- Jorgensonian (neoclassical) theory of investment
- Tobin's Q theory of investment
- The "neoclassical synthesis": investment with convex adjustment costs

The Jorgensonian neoclassical theory of investment

- Continuous time.
- Neoclassical firm which maximizes the present discounted (at the instantaneous risk-free interest rate r) value of profits over its lifetime T, where T can be ∞.
- Production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \ 0 < \alpha \leq 1$$

  A_t is TFP and K_t and L_t capital and labour inputs.
- Competitive product and factor markets. w_t and r_t^K are the real wage and rental price of capital.

Firm's problem

- The firm's optimization problem is

$$\max_{K_t, L_t} V_0 = \int_0^\infty \left[A_t K_t^\alpha L_t^{1-\alpha} - w_t L_t - r_t^K K_t \right] e^{-rt} dt \quad (154)$$

with FOCs

$$(1-\alpha) \left(K_t / L_t \right)^\alpha = w_t \quad (155)$$

$$\alpha A_t \left(K_t / L_t \right)^{\alpha - 1} = r_t^K . \quad (156)$$

- Given zero costs of adjusting labour and capital, the firm chooses input quantities to equate the marginal product of each factor to its rental cost period-by-period.

- Equation

$$\alpha A_t \left(K_t/L_t\right)^{\alpha-1} = r_t^K.$$

  embodies Jorgenson's (1963) neoclassical theory of investment.
- The return to one unit of capital invested in the firm is its marginal product and at an optimum this must equal its rental price.

What if capital is bought rather than rented?

- Most capital is not rented.
- What is the equivalent per period cost of a unit of capital if capital can be scrapped at no cost and resold?
- The opportunity cost (in units of output) of a unit of capital costing $p^K t$ is the sum of:
  - the interest rate foregone rp_t^K
  - the depreciation cost δp_t^K
  - the capital loss/gain $\dot{p}_t^K$
- Hence, the rental price equivalent (the user cost) is

$$r_t^K = \left[r + \delta - \frac{\dot{p}_t^K}{p_t^K} \right] p_t^K. \tag{157}$$

Summarizing

For the Jorgensonian theory of investment:

1. The cost of capital depends on the instantaneous interest rate. In general, the relevant interest rate is the one which applies for the period over which capital can be adjusted;
2. It's irrelevant whether the firm rents or owns its capital.[1]. The firm should invest up to the point where the marginal cost of capital equals its user cost.
3. Remembering that with Cobb-Douglas technology $MPK = \alpha APK$, the optimal investment condition can be written as

$$\alpha \frac{Y_t}{K_t} = r_t^K. \tag{158}$$

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[1] In fact, the user cost in (157) is the price at which the firm can rent capital for one instant if the rental market for capital is competitive.

Implications (logical problems) of the theory I

The absence of adjustment costs implies that:

1. The firm does not have to look into the future. The future is irrelevant for investment decisions $\rightarrow$ clearly counterfactual.
2. The stock of capital K_t jumps discretely to keep the marginal product of capital equal to its user cost $\rightarrow$ Gross investment $\dot{K}_t$ $\rightarrow$ investment rate is zero in the absence of shocks and +/- infinity otherwise.[2].
3. The theory does **not** predict that investment is negatively related to the user cost of capital. In fact it is not a theory of investment but rather of the stock of capital

[2] This would improve only slightly if time were discrete. Firms would still want to do all their adjustment immediately after a shock and be inactive otherwise. The volatility of investment would still be counterfactually high.

Implications (logical problems) of the theory II

4. It is not even a theory of the capital stock K_t. K_t cannot be determined separately from L_t or Y_t (which are undetermined under CRS). $\rightarrow$ theory of the optimal capital/labour (or capital/output) ratio. Thus, it is a theory of the optimal capital stock *conditional* on the level of output.
5. The interest rate is assumed to be exogenous. What happens in general equilibrium?

Early empirical tests

- Hall and Jorgenson (1967) sidestepped the problem associated with 3. by assuming that
  - output is predetermined when the firm chooses the stock of capital
  - the capital stock cannot adjust immediately to its frictionless target in (158); e.g. $\Delta K_t = \sum_{\tau=0}^{n} \beta_\tau \Delta K^*_{t-\tau}$.
- This led to estimating an equation of the form

$$I_t - \delta K_t = \sum_{\tau=0}^{n} \beta_\tau \Delta K^*_{t-\tau} \tag{159}$$

Replacing in (159) gives

$$I_t - \delta K_t = \sum_{\tau=0}^{n} \left(\beta_\tau \alpha \Delta \frac{Y_{t-\tau}}{r^K_{t-\tau}} \right) \tag{160}$$

Findings

- From their estimates of $\beta_\tau \alpha$, Hall and Jorgenson (1963) concluded that the data supported the theory.
- Yet, since they impose that Y_t and r_t^K enter with the same elasticity the coefficients would be sizeable and significant even if only output but not the user cost of capital were empirically relevant.
- Eisner (1969) shows that once $Y_{t-\tau}$ and $r_{t-\tau}^K$ enter separately the estimated coefficient on $r_{t-\tau}^K$ were extremely small, casting serious doubts on the user cost of capital being an important determinant of investment.

Remarks

- It has to be noted that the above formulation is *not* an appropriate to test of the Jorgensonian theory of the optimal capital stock alone.
- By imposing the short run adjustment dynamics (159) it tests two hypotheses at the same time: the correctness of the dynamics specification and the adequacy of the Jorgensonian theory. This is a very important point to understand its empirical failure.
- From a theoretical point of view, the estimating equation (159) introduces adjustment costs *implicitly* without having explicitly modelled them and derived their theoretical implications.

Tobin's q theory of investment

Tobin (1969) introduced an alternative theory of investment.

- He defined the quantity Q – the ratio between the stock market value of a firm V_t and the resale value of its capital stock at market prices. [3].
- So, Tobin's *average* Q is given by

$$Q_t = \frac{V_t}{p_t^K K_t}. \tag{161}$$

- Tobin argued that investment should be an increasing function of Q_t.

[3] In our setup, the stock market value of the firm (ruling out bubbles in stock prices) is given by the present discounted value of the firm's profits on an optimal path; see equation (154)

Empirical tests

- It is effectively a no-arbitrage argument. If Q_t is below one, a profit can be made by buying the firm on the stock market at V_t, scrapping its capital and selling it at $p_t^K K_t > V_t$. Viceversa, if Q_t is larger than one.
- Unfortunately, as for the user cost of capital, the coefficient on Q_t in estimated investment equations turned out to be disappointingly small.

The “neoclassical synthesis”

- In the early 80s researchers - namely Abel (1979) and Hayashi (1982) - were able to unify these two strands of the theory. The model they used encompasses Jorgenson’s model by allowing for positive adjustment costs.
- This model (which was first introduced in the late 60s but not linked to Tobin’s Q at the time) has the great advantage of explicitly introducing adjustment costs in the optimization problem.
- Rather than assuming that capital can be costlessly installed and scrapped the model assumes convex costs of investment and disinvestment e.g. $C(I_t) = cI_t^2/2$.

Implications

1. The existence of *adjustment costs* implies that expectations are important. $\rightarrow$ When investing today the likelihood that the investment decision has to be reversed tomorrow due to a fall in productivity matters.
2. The *convexity of the cost of investment* implies that the marginal investment cost (and the average unit cost) is increasing in the size of investment/disinvestment. So, it is optimal to spread investment over time.

Investment under convex adjustment cost

- Output and capital are the same good and no depreciation (just for simplicity).
- The firm can freely borrow and lend at the instantaneous market interest rate r.
- Its optimization problem is

$$\max_{\{L_t, I_t, K_t\}_{t=0}^{T}} V_0 = \int_0^T \left[A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - I_t \left(1 + \frac{cI_t}{2} \right) \right] e^{-rt} dt$$

$$\text{s.t. } K_0 \text{ given}, K_t \geq 0 \tag{162}$$

$$\dot{K}_t = I_t \tag{163}$$

Investment under convex adjustment costs II

The Lagrangean associated with the above problem is

$$\mathfrak{L}_0 = \int_0^T \left[A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - I_t \left(1 + \frac{c I_t}{2} \right) + q_t \left(I_t - \dot{K}_t \right) \right] e^{-rt} dt,$$

which has been written in such a way that q_t, the Lagrange multiplier associated with constraint (163) at each time t, is positive.

q_t **captures the marginal benefit (the marginal increase in profits) at time** t **of an exogenous marginal increase in** K_{t+dt}. **For this reason it is called the shadow price of capital.**

Investment under convex adjustment costs III

Integrating the term in $\dot{K}_t$ by parts the Lagrangean can be written as

$$\int_0^T \left[A_t K_t^{\alpha} L_t^{1-\alpha} - w_t L_t - I_t \left(1 + \frac{cI_t}{2} \right) + q_t I_t + \dot{q}_t K_t - r q_t K_t \right] e^{-rt} dt$$

$$+ q_0 K_0 - q_T K_T e^{-rt}.$$

The necessary conditions for an optimum are

$$\frac{\partial \mathfrak{L}_0}{\partial L_t} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \tag{164}$$

$$\frac{\partial \mathfrak{L}_0}{\partial I_t} = -1 - cI_t + q_t = 0 \tag{165}$$

$$\frac{\partial \mathfrak{L}_0}{\partial K_t} = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} + \dot{q}_t - r q_t = 0 \tag{166}$$

$$\frac{\partial \mathfrak{L}_0}{\partial K_T} \rightarrow q_T \geq 0,\ \ K_T \geq 0 \text{ with complementary slackness} \tag{167}$$

$$\text{or equivalently } q_T K_T e^{-rT} = 0. \tag{168}$$

Investment under convex adjustment costs IV

- Equations (164)-(168), together with equations (162) and (163), fully characterize the path for I_t and K_t.
- Equation (165) requires the shadow price of capital q_t at an optimum to equal its price of 1 plus the marginal installation cost.
- The intertemporal element comes from equation (166) which describes the evolution of the shadow price of capital. Optimal intertemporal capital allocation requires the shadow return on capital $\left(MPK_t + \dot{q}_t\right)/q_t$ to equal the interest rate r.

Transversality condition

- The transversality condition (168) is easier to interpret in finite time. If time is finite it implies $q_T K_T = 0$. On an optimal path, at the terminal date either all capital is consumed or if some of it is left - $K_T > 0$ - its value must be zero.
- If $T \to \infty$ it becomes $\lim_{t\to\infty} q_t K_t e^{-rt} = 0 \to$ q_t must not grow at rate faster than r if $\lim_{T\to\infty} K_T > 0$.
- $K_t > 0$ always, on an optimal path[4] Thus,

$$\lim_{t\to\infty} q_t e^{-rt} = 0, \tag{169}$$

[4] On an optimal path K_t cannot converge to zero as $t \to \infty$. You can check that as $I_t \to 0$ (which is the case on a path approaching the steady state) the value of the firm V_0 becomes infinitely negative as K_t goes to zero.

Solving the model

In what follows we assume $T = \infty$.

- We can rearrange equation (165) to write

$$I_t = c^{-1}\left(q_t - 1\right). \tag{170}$$

- Replacing for I_t in (163) one obtains

$$\dot{K}_t = c^{-1}\left(q_t - 1\right). \tag{171}$$

- If we assume, with little loss of generality, that the labour force is constant and equal to 1, equation (171) together with (166), (169) and the initial level of capital K_0 fully characterize the evolution of K_t and q_t on the optimal path.

Solving the model II

- Equation (170) implies that given the current capital stock K_0, the *only* determinant of investment is q_t, the shadow value of capital. Investment is positive if q_t exceeds one, negative if below[5].
- q_t can be obtained by multiplying both sides of equation (166) by e^{-rt} and integrating forward. This gives

$$\int_0^\infty (\dot{q}_t - rq_t)\, e^{-rt} dt = \int_0^\infty \frac{dq_t e^{-rt}}{ds} dt = -\int_0^\infty \left(\alpha A_t K_t^\alpha L_t^{1-\alpha}\right) e^{-rt} dt$$

and using equation (169)

$$q_0 = \int_0^\infty \left(\alpha A_t K_t^\alpha L_t^{1-\alpha}\right) e^{-rt} dt. \tag{172}$$

Marginal q

- The shadow value of capital equals the discounted value of the present and future return (in terms of production and lower adjustment costs) of the *marginal* unit of capital on an optimal path divided by its price, which is one.
- For this reason, q_t is also known as marginal q, as it is the counterpart - at the margin - of Tobin's Q[6]. Marginal q is not observable since it depends on future values of A_t and K_t and L_t.

Continued

For the analysis of the model, steady state, saddle path, response to changes in exogenous variables see Romer.